Week 13 Solutions

(13-1) Use the formula relating p, V, N, k_B and T, which is the equation of state for the gas,

$$pV = Nk_BT$$

or,
$$\frac{N}{V} = \frac{p}{k_BT} = \frac{10^{-11}\ \text{mm Hg}}{(1.38\times10^{-23}\ \text{J/K})(300\ \text{K})}$$

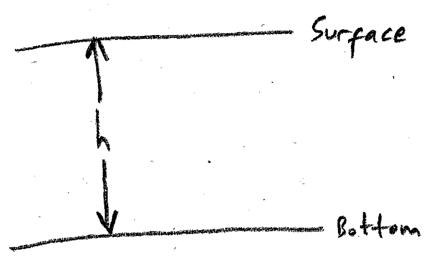
$$= \frac{10^{-11} \times 133.32\ \text{Pa}}{(1.38\times10^{-23}\ \text{J/K})(300\ \text{K})}$$

$$= 3.22 \times 10^{11}\ \text{\# molecules}/\text{m}^3$$

$$= 3.22 \times 10^{5}\ \text{\# molecules}/\text{cm}^3.$$

(13-2) At the bottom of the lake; $P_B V_B = nRT_B$.
At the surface, $P_S V_S = nRT_S$.
The relation between P_B and P_S is

$$P_B = P_S + \rho g h.$$



From the first two equations, we have

$$\frac{P_B V_B}{P_S V_S} = \frac{T_B}{T_S} \implies P_B = P_S \frac{V_S}{V_B}\frac{T_B}{T_S}$$

so,

$$P_S \frac{V_S}{V_B}\frac{T_B}{T_S} = P_S + \rho g h$$

$$\frac{V_S}{V_B}\frac{T_B}{T_S} = 1 + \frac{\rho g h}{P_S}$$

$\rho = 1000\ \text{kg/m}^3$ for water
P_S = atmospheric pressure, 1 atm, or 101.325 kPa

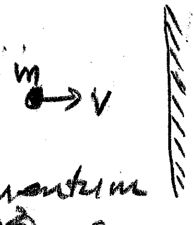
$$V_S = V_B \frac{T_S}{T_B}\left(1 + \frac{\rho g h}{P_S}\right)$$

$$= (1\ \text{cm}^3)\frac{293\ \text{K}}{288\ \text{K}}\left(1 + \frac{(1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(2\ \text{m})}{101.325\times10^3\ \text{Pa}}\right)$$

$$= 1.21\ \text{cm}^3$$

(13-3) The average velocity of the molecules is zero because there is a great number of molecules, where their velocities are randomly oriented to any direction. When we average them out, we will get zero.

(13-4) The initial kinetic energy is $\frac{1}{2}mv^2$. Since the collision is elastic, total kin. Energy is conserved. Wall picks up momentum but since its mass is very large $\Delta K_W = \frac{(\Delta p_W)^2}{2M_W}$, $\Delta K_W = 0$, so the atom's kinetic energy is still $\frac{1}{2}mv^2$, but the velocity is opposite.



(13-5) i) Since the kinetic energy of the molecules is related to temperature only, $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}k_BT$, so it means that both atoms (He and Krypton) have the same kinetic energy.

ii) Since $V_{rms} = \sqrt{3k_BT/m}$, then it means that the r.m.s. speed of Helium is higher than the r.m.s. speed of Krypton by the factor $\sqrt{\frac{m_K}{m_{He}}} = \sqrt{\frac{84\times1.6\times10^{-27}\ kg}{4\times1.6\times10^{-27}\ kg}} = \sqrt{21} \sim 4.6$

(13-6) The equation of state is $pV = nRT$. Let's double the pressure but keep the volume. So, $(2p)V = nRT'$, where T' is the new temperature after we change the state of the gas. Divide the second equation by the first one, we have

$$2 = \frac{T'}{T} \Longrightarrow T' = 2T = 2\times(273+20)$$

$$= (2\times293)\,K. = 586K$$

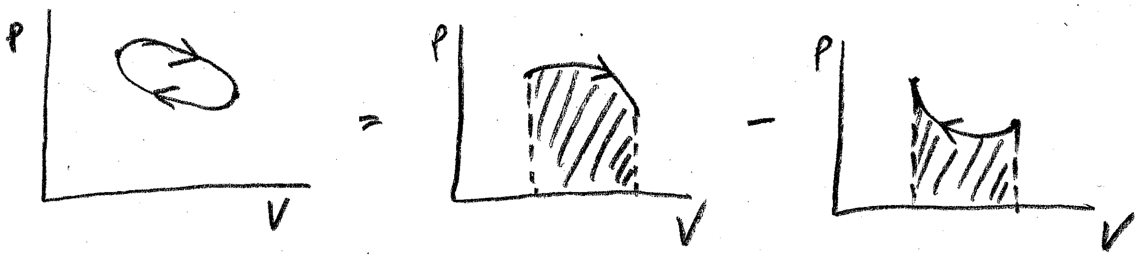
Since $v_{rms} = \sqrt{\frac{3k_BT}{m}}$, the r.m.s. speed of the particles increases by the factor of $\sqrt{\frac{T'}{T}} = \sqrt{2}$.

(13-7) The work done can be calculated from the area under the curve.

i) So the largest work done is in the process $A \to B \to D$.

ii) And the least work done is in the process $A \to C \to D$.

(13-8) i)



So, the work done in going around the loop is equal to the area of the loop.

ii)

(13-9) i) Since the volume doesn't change, the work done is zero.

ii) $\Delta U = \frac{3}{2} nR\Delta T = \frac{3}{2}$ (2 mols)(8.314 J.K^{-1}.mol^{-1})(586 K − 293 K)

$= 7308\ J$

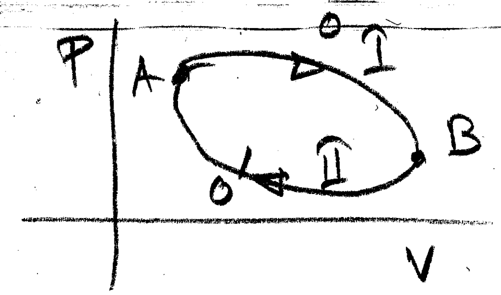
(13-10) **Mechanical Equivalent**: 4.18 J of mechanical work mimics 1 cal of Heat.

Heat Exchange of Energy due to temperature Difference.

Both involve interaction with outside, both are path dependent.

INTERNAL ENERGY Energy which resides within the system, change is path independent $U_{MA} = \frac{3}{2} N k_B T$

13-11



Since internal energy does not depend on the path and depends only on the end points,

$$dU_{A\to o\to B} = U_B - U_A$$

$$dU_{B\to o'\to A} = U_A - U_B$$

$$= -dU_{A\to o\to B}$$

$$\therefore\ dU_{B\to o'\to A} + dU_{A\to o\to B} = 0$$

or $dU_I + dU_{II} = 0$

$$\therefore\ dU_{A-o-B-o'-B}$$

$$= dU_{cycle} = dU_{A\to o\to B} + dU_{B-o'\to A}$$

$$= dU_I + dU_{II}$$

$$= 0$$

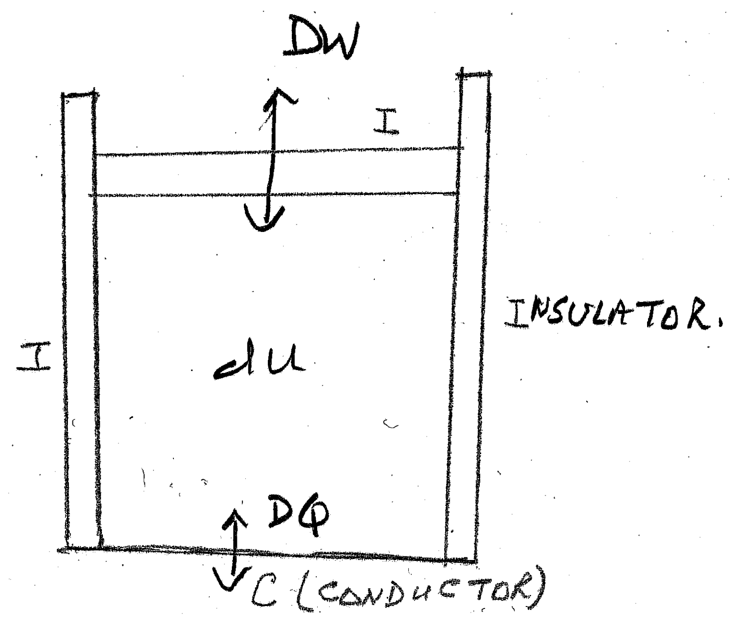
$$DQ\Big|_{cycle} = dU\Big|_{cycle} + DW\Big|_{cycle} \Rightarrow DQ\Big|_{cycle} = DW\Big|_{cycle}$$

as $dU_{cycle} = 0$

But we have seen that $DW\big|_{cycle} > 0$

$\therefore\ DQ\Big|_{cycle} > 0$ So we need to add heat!

13-12. The picture shows a thermodynamic system consisting of a certain amount of gas. Apart from the bottom (C) all the walls and piston are insulators (I). Discuss the three modes in which the gas can change its energy indicating clearly which of the changes are path dependent and which change is intrinsic to the gas?



1. System can exchange heat DQ with its sorroundings through the conducting bottom (DQ)

2. System can move piston and exchange work with the outside (DW)

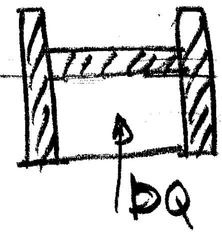
3. System can change its internal energy (dU)

Conservation of Energy law:

$$\pm DQ \pm DW \pm dU = 0$$

13-13 Constant Volume (isochoric)

i) $V = \text{const}$



$DW = 0$

$DQ = dU$

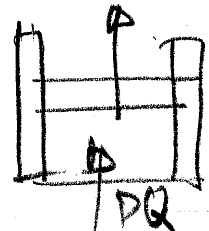
$= nC_v \Delta T$

$DW = 0$ as piston doesn't move

This Energy exchange is intrinsic as dU is not path dependent. So $DQ = dU$ = path independent

ISOBARIC

ii) Constant pressure ($p = \text{const}$)



Piston moves

$DW \neq 0$

$DW = p\Delta V$

$DQ = dU + p\Delta V$

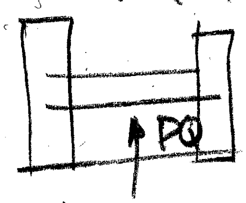
$= nC_v\Delta T + p\Delta V$

This heat exchange is not intrinsic

iii) ISOTHERMAL (constant temperature)

$T = \text{const}$

$DQ = DW$



$DU = 0$ as $\Delta T = 0$

$(nC_v\Delta T)$

$\Delta T = 0 \Rightarrow dU = 0$

(piston moves)

$DQ = 0 + DW$

This is also not intrinsic

13-14 Solids and liquids have fixed volumes and hence All processes for them are constant volume processes. However gases do not have fixed volumes and sp. heat very much depends on process. So we donot get a unique C by writing as $\frac{\Delta Q}{m\Delta T}$