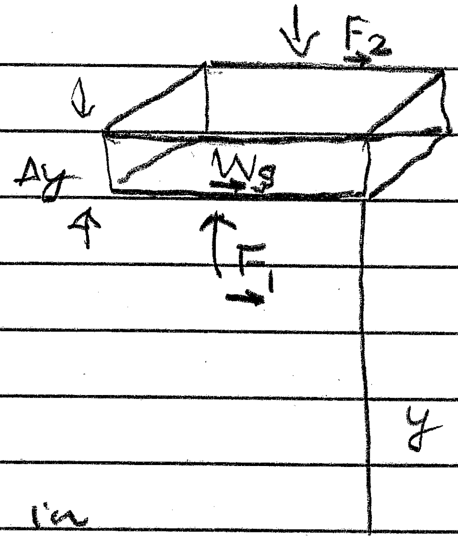
Solutions – Wk12



12-1 Consider a layer of Area A and thickness Δy lying at a height y above zero. If it is to be in equilibrium the sum of all the forces acting on it must be zero.

There are three forces acting on it

$$\vec{F}_1 = +P(y)\,A\,\hat{y} \quad \text{(fluid from below pushing up)}$$

(density d) $$\vec{W}_g = -d\,g\,A\,\Delta y\,\hat{y} \quad \text{(weight of fluid of density d)}$$

$$\vec{F}_2 = -P(y+\Delta y)\,A\,\hat{y} \quad \text{(fluid from above pushing down on it)}$$

Hence

$$\vec{F}_1 + \vec{F}_2 + \vec{W}_g = 0$$

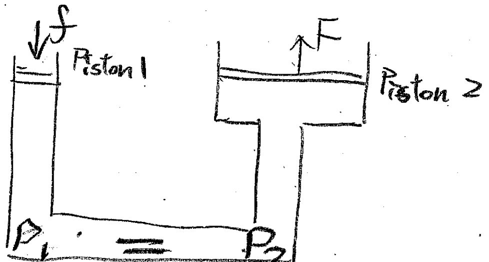
$$P(y)\,A - P(y+\Delta y)\,A - d\,g\,A\,\Delta y = 0$$

$$\therefore\ P(y+\Delta y) - P(y) = -d\,g\,\Delta y.$$

If Δy is very small this result will be good for both gases and liquids.

However, if Δy is large it will work only for liquids because ρ_{Liq} is independent of pressure while ρ_{gas} depends on pressure

12-2.

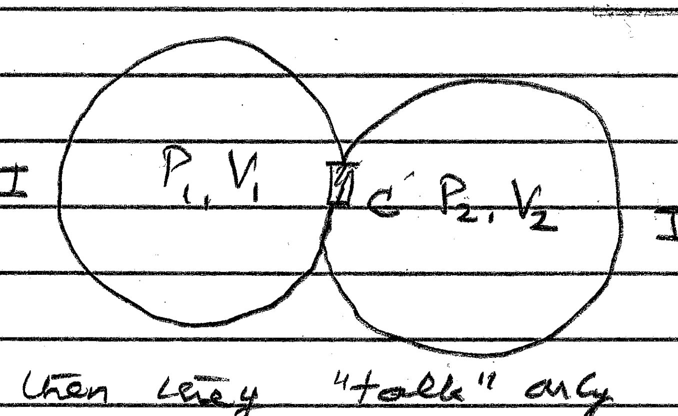


From prob. 1, we can see that the pressure of a fluid depends on its height. Thus, if the two pistons are at the same height, the corresponding pressure below the pistons should also be the same. That is

$$P_1 = P_2 \Rightarrow \frac{f}{A_1} = \frac{F}{A_2}, \quad \text{and} \quad \frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \frac{\pi\left(\frac{d_1}{2}\right)^2}{\pi\left(\frac{d_2}{2}\right)^2} = \left(\frac{d_1}{d_2}\right)^2 \qquad (F = 45000\ N)$$

$$\Rightarrow F = \frac{A_2}{A_1} f = \left(\frac{d_2}{d_1}\right)^2 f = \left(\frac{30\,cm}{1\,cm}\right)^2 \times 50\,N = 900 \times 50 = 45000\ N$$

12-3 If the systems are isolated from the surroundings but have a conducting (solid) wall in between then they "talk" only to one another. If one changes the other must. We can label their starting states by their pressures and volumes, respectively (P_1, V_1) and (P_2, V_2).



Once they are put together two observations are possible:

I Neither of them changes although $P_1 \neq P_2$ and $V_1 \neq V_2$

II Both of them change $P_1 \to P_1'$, $V_1 \to V_1'$, $P_2 \to P_2'$, $V_2 \to V_2'$ but if we are patient all changes stop even though

$$P_1' \neq P_2' \text{ and } V_1' \neq V_2'.$$

What can we conclude.

1. If there is no change then the two systems must be in equilibrium. This is a new kind of equilibrium (not mechanical, see below)
2. Pressure and Volume are irrelevant to define this new equilibrium.
3. We must attach another label to our systems whose equality will ensure equilibrium. That new label is called TEMPERATURE (θ) and that is why this is called thermal equilibrium. Indeed the definition of temperature is that two systems can be in Thermal Equilibrium if and only if they have the same temperature

A direct implication of this is that a single system can be in thermal equilibrium only if temperature is the same at all points in it.

12-4.

The relation between the two scales is given by

$$T(°F) = \frac{9}{5}T(°C) + 32$$

If the two scales have a common numerical value "n", then

$$T(°F) = T(°C) = n$$

$$\Rightarrow \quad n = \frac{9}{5}n + 32$$

$$\Rightarrow \quad n = -40$$

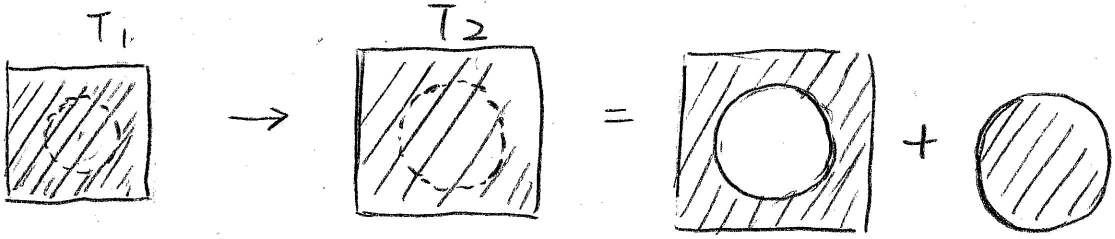
That is, $\underline{-40°C = -40°F}$. #

12-5.

- Heat is the energy which is exchanged between two systems when they are at different temperatures and there is a conducting wall between them.

- Specific heat involves change of temperature, it is the quantity of heat which must be transferred to/from a system of unit mass to change its temperature by one degree. Latent heat is involved when there is a change of state (solid $\leftrightarrow$ liquid. liquid $\leftrightarrow$ gas) without change of temperature.

12-6



Since each part of the plate would expand with respect to the same expansion coefficient α, the expansion of the hole can be viewed as that of the circular plate. (see fig. above), Thus, the final radius of the hole would be

$$R_f = R_o [1 + \alpha(\theta - \theta_o)]$$

$$\theta_o = 20°C$$

On the other hand, in order to let the sphere pass through the hole, its final radius r_f should be the same as the radius of the hole. $(R_f = r_f)$

$$r_f = r_o [1 + \alpha_{cu}(\theta - \theta_o)]$$

$$\because r_f = R_f$$

$$\Rightarrow R_o [1 + \alpha_{Al}(\theta - \theta_o)] = r_o [1 + \alpha_{Cu}(\theta - \theta_o)]$$

$$\Rightarrow R_o - r_o = (r_o \alpha_{cu} - R_o \alpha_{Al})(\theta - \theta_o)$$

$$\Rightarrow (\theta - \theta_o) = \frac{R_o - r_o}{r_o \alpha_{cu} - R_o \alpha_{Al}} = \frac{1.999\ cm - 2\ cm}{[(2\ cm) \times 17 \times 10^{-6}\ C^{-1} - (1.999\ cm) \times 24 \times 10^{-6}\ C^{-1}]}$$

$$= 71.55\ °C$$

$$\Rightarrow \theta = 91.55\ °C$$ #

12-7. The expansion coefficient for the beaker volume is given by

$\beta_{glass} = 3\alpha_{glass} = 3 \times 3.2 \times 10^{-6}\ C^{-1} = 9.6 \times 10^{-6}\ C^{-1}$ [Vol. Expansion hence 3α]

As for the water, we have

$$\beta_{water} = 2.1 \times 10^{-4}\ C^{-1}$$

Since the β_{water} is larger than β_{glass}, if we increase the temperature, some of the water will spill out. ($\because$ initially, $V_{o,water} = V_{o,glass} = V_o$)

12-7.

Thus, the volume of the spilled water is

[$V_0 = 100\ cm^3$]

$V_{spill} = V_{water} - V_{glass}$

$= V_0[1+\beta_{water}\Delta T) - V_0(1+\beta_{glass}\Delta T)$

$= V_0 \Delta T(\beta_{water} - \beta_{glass}) = (100\ cm^3) \times (90-20) \times (2.1\times10^{-4} - 9.6\times10^{-6})$

$= 1.4\ \ cm^3$ #

12-8 Since, energy is conserved,

According to the 1st law of thermodynamics, the heat released from the lead ball is the same as the heat absorbed by the calorimeter and liquid. That is

$-\Delta Q_{Pb} = \Delta Q_{liq} + \Delta Q_{cal}$, $\Rightarrow -m_{Pb}C_{Pb}(T_f - T_i') = m_{liq}C_{liq}(T_f - T_i'') + m_{cal}C_{cal}(T_f - T_i'')$

T.

$\Rightarrow -(0.25\ kg)(130\ J/kg°c)(25-210) = (0.2\ kg) \times C_{liq}(25-20)$
$+ (0.09) \times (900\ J/kg c) \times (25-20)$

$\Rightarrow \underline{C_{liq} = 5607.5\ J/kg°C}$ #

12-9. Like Prob. 8, we have

[But now ice has to melt so Latent Heat is involved].

$-\Delta Q_{water} = \Delta Q_{ice}$

Assume that the system reaches a common temperature T.

$\Rightarrow -m_{water}C_{water}(T-20) = m_{ice}C_{ice}[0-(-5)] + L_{ice}m_{ice}$
$+ m_{ice}C_{water}(T-0)$

$-(m_w + m_{ice})C_W T = 5m_{ice}C_{ice} + L_{ice}m_{ice} - 20m_W C_W$

$$T = \frac{5\times0.02\times2000 + 334\times10^3\times0.02 - 20\times0.1\times4186}{-(0.12)\times4186}$$

$= 2.97\ °C$ #

12-10

- Conduction is the transfer of thermal energy layer by layer via physical material, and it operates in solid and stationary liquids and gases based on direct physical contact between two layers
- Convection is the transfer of heat by the motion of a fluid. and it operates in liquids and gases based on thermal stirring.

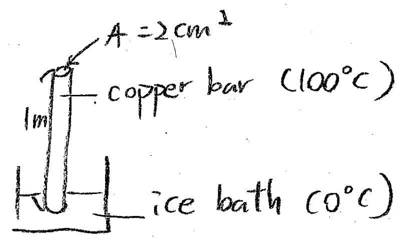
12-11

$$\frac{DQ}{\Delta t} = -kA\frac{\Delta T}{\Delta X}$$

Here the negative sign means that a heat current always goes against the temperature gradient, because heat always flows from a point at a high temperature to a point at a low temperature.

12-12.

We have here the rate equation for conduction:



$[2\,cm^2 = 2\times10^{-4}\,m^2]$

$$\frac{DQ}{\Delta t} = -kA\frac{\Delta T}{\Delta X}$$

and we know that

$A = 2\,cm^2$, $\Delta X = 1\,m$, $\Delta T = 100\,°C$

$k = 39\,J/m\text{-}s\text{-}°C$, $\Delta t = 10\,s$

$$\Rightarrow \quad DQ = -kA\,(\Delta T/\Delta x)\times\Delta t$$
$$= -(39\,J/m s °C)\times(2\times10^{-4}m^2)\times\left(\frac{100°C}{1m}\right)\times 10\,s$$
$$= -7.8\,J$$

In order to let m kg ice melt, the needed heat is given by

$$\Delta Q = L_{ice}\,m \Rightarrow m = \frac{\Delta Q}{L_{ice}} = \frac{7.8\,J}{(334\times10^3\,J/kg)} = 2.33\times10^{-5}\,kg$$
$$= 2.33\times10^{-2}\,g \quad \#$$

2.33×10^{-2} g of ice will melt due to the conduction of heat.

12-13

For the smaller sphere, the heat radiation rate is given by

$$\frac{DQ_1}{\Delta t} = A_1 e \sigma T_1^2 \quad , \quad A_1 = 4\pi\left(\frac{d_1}{2}\right)^2 \leftarrow \text{surface area } 4\pi R^2 \quad , \quad d_1 = 1m$$

Likewise, for the larger sphere, we have

$$\frac{DQ_2}{\Delta t} = A_2 e \sigma T_2^2 \quad , \quad A_2 = 4\pi\left(\frac{d_2}{2}\right)^2 \quad , \quad d_2 = 4m$$

(i) If they are at the same temperature, then

$$T_1 = T_2 = T$$

$$\left[\frac{\left(\frac{DQ_1}{\Delta t}\right)}{\left(\frac{DQ_2}{\Delta t}\right)}\right] = \frac{A_1 e \sigma T^2}{A_2 e \sigma T^2} = \frac{A_1}{A_2} = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

$\Rightarrow$ A_2 emits more heat radiation per second. $\frac{DQ_2}{\Delta t}$ is larger than $\frac{DQ_1}{\Delta t}$ by the factor of 16.

(ii) If they have to emit the same amount of heat per sec., then

$$\frac{DQ_1}{\Delta t} = \frac{DQ_2}{\Delta t} \Rightarrow A_1 \cancel{e}\cancel{\sigma} T_1^2 = A_2 \cancel{e}\cancel{\sigma} T_2^2$$

$$\Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{A_2}{A_1}} = \sqrt{\left(\frac{4}{1}\right)} = 2$$

$\Rightarrow$ We need to increase the temperature of T_1 by the factor of 2. because $\frac{DQ}{\Delta t} \propto T^4$.

12-14.

We have

$$\frac{DQ}{\Delta t} = A e \sigma (T_1^4 - T_2^4)$$
$$= A e \sigma (T_1 - T_2)(T_1 + T_2)(T_1^2 + T_2^2)$$

If $(T_1 - T_2) \ll T_1$ or T_2, then $T_1 \simeq T_2$. That is, for small temperature, $(T_1 + T_2)$, $(T_1^2 + T_2^2)$ are approximately constant, so the rate of cooling is proportional to the temperature difference.