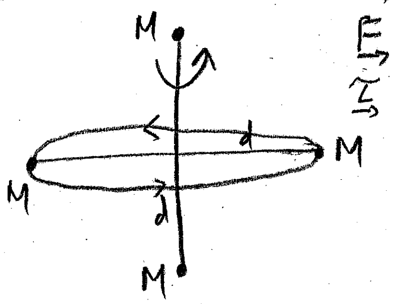
Week 11 Solutions

(11-1) Rigid body is a system that consists of many mass points (m_i) located at different points ($\underset{\rightarrow}{r_i}$) but ($\underset{\rightarrow}{r_i} - \underset{\rightarrow}{r_j}$) is fixed so it neither changes shape nor size as it moves. IN TRANSLATION ALL POINTS HAVE SAME VELOCITY AND SAME LINEAR ACCELERATION. IN ROTATION ALL POINTS HAVE SAME ANGULAR VELOCITY AND ANGULAR ACCELERATION

(11-2) Force is a physical agency which is necessary to cause linear acceleration, while torque is a physical agency which is necessary to cause angular acceleration and hence rotation about an axis. In the plane perpendicular to the axis, go a distance $\underset{\rightarrow}{r}$ from the axis. Apply a force $\underset{\rightarrow}{F}$ perpendicular to $\underset{\rightarrow}{r}$ to cause a torque $\underset{\rightarrow}{\tau} = [\underset{\rightarrow}{r} \times \underset{\rightarrow}{F}]$ along the axis.

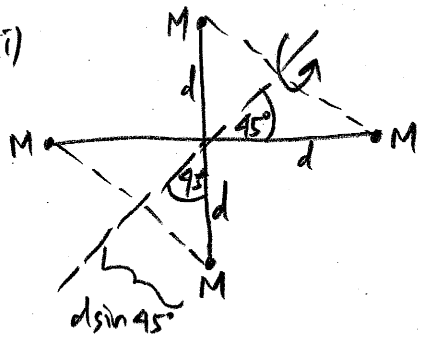
(11-3) (i)



$I = Md^2 + Md^2 = 2Md^2$

is Moment of Inertia about Y-axis

(ii)



$\sin 45 = \frac{1}{\sqrt{2}}$

The moment of inertia I about the dashed line is

$$I = M(d \sin 45°)^2 + M(d \sin 45°)^2 + M(d \sin 45°)^2 + M(d \sin 45°)^2$$

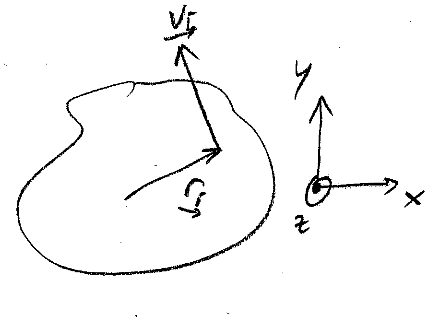
$$= 4M(d \sin 45°)^2$$

$$= 4M\left(d \tfrac{1}{2}\sqrt{2}\right)^2$$

$$= 4M d^2 \tfrac{1}{4} \cdot 2$$

$$= 2Md^2$$

(11-4) The angular momentum of a rigid body is



$$\underset{\rightarrow}{L} = \sum_i m_i \underset{\rightarrow}{r_i} \times \underset{\rightarrow}{v_i}$$

$$= \sum_i m_i \underset{\rightarrow}{r_i} \times (r_i \omega \hat{v}) = \sum_i m_i r_i^2 \omega (\hat{r}_i \times \hat{v})$$

$$\underset{\rightarrow}{L} = \sum_i m_i r_i^2 \underset{\rightarrow}{\omega} = I \underset{\rightarrow}{\omega}$$

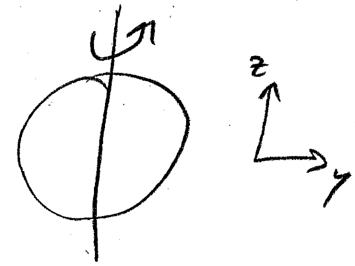
$K = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i (r_i \omega \hat{v})^2$

$= \sum_i \frac{1}{2} m_i r_i^2 \omega^2$

$= \sum_i \frac{1}{2} m_i r_i^2 \left(\frac{L}{I}\right)^2 = \frac{1}{2} I \left(\frac{L}{I}\right)^2$

$= \frac{L^2}{2I}$

(11-5) First, calculate the angular speed of the Earth.



$\vec{\omega} = \frac{2\pi}{T} \hat{z}$, where $T = 24$ hours.

Then, calculate its moment of inertia.

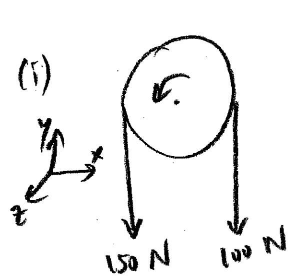
$I = \frac{1}{3} MR^2$

Therefore, its kinergy is $K = \frac{1}{2} I \omega^2 = \frac{1}{2} \cdot \frac{1}{3} MR^2 \cdot \left(\frac{2\pi}{T}\right)^2$

$= \frac{1}{2} \cdot \frac{1}{3} (5.9742 \times 10^{24}\ kg)(6300 km)^2 \left(\frac{2\pi}{24 \times 60 \times 60\ sec}\right)^2$

$= 2.1 \times 10^{29}$ Joule.

(11-6) (i)



$\sum \vec{\tau} = \{(150\ N)(0.5\ m) - (100\ N)(0.5\ m)\} \hat{z}$

$= 25\ Nm\ \hat{z}$

(ii) $I = \frac{1}{2} Mr^2 = \frac{1}{2} (5\ kg)(0.5\ m)^2 = 0.625\ kg \cdot m^2$

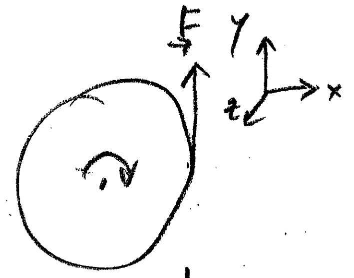
$\sum \vec{\tau} = I \vec{\alpha}$

$25\ Nm\ \hat{z} = (0.625\ kg\ m^2)\ \vec{\alpha}$

$\vec{\alpha} = 40\ s^{-2}\ \hat{z}$ (Note that $N = kg \cdot m/s^2$)

(rad/s^2)

(11-7)



$\vec{\omega} = -2 \frac{rad}{s} \hat{z}$

$r = 0.2\,m.$

$\sum \vec{\tau} = I \vec{\alpha}$, where $\vec{\alpha} = \frac{0 - \vec{\omega}}{\Delta t}$

$= \frac{2 \frac{rad}{s} \hat{z}}{5\ s}$

$= 0.4\ rad/s^2\ \hat{z}$

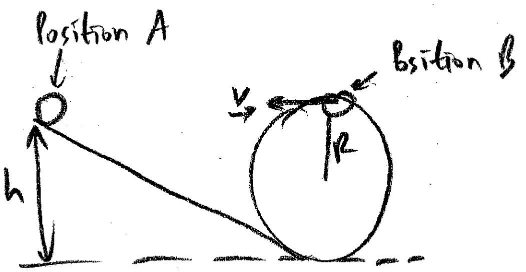
$\sum \vec{\tau} = \frac{1}{2} m r^2 \vec{\alpha}$

$= \frac{1}{2}(0.05\ kg)(\frac{20}{100}\ m)^2 (0.4\ \frac{rad}{s^2})\ \hat{z}$

$= 4 \times 10^{-4}\ Nm\ \hat{z}$

$F = \frac{\tau}{r} = \frac{4 \times 10^{-4}}{0.2} N = 2 \times 10^{-3} N$

(11-8)



Use the conservation of energy between position A and B.

$mgh = mg(2R) + \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

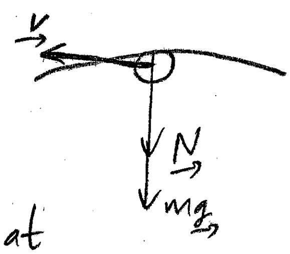
where $v = r\omega$ and $I = \frac{2}{5} m r^2$.

$mgh = 2mgR + \frac{1}{2} m v^2 + \frac{1}{2} \cdot \frac{2}{5} m r^2 (\frac{v}{r})^2$

$= 2mgR + \frac{7}{10} m v^2$

$gh = 2gR + \frac{7}{10} v^2 \Rightarrow v^2 = \frac{10}{7}(gh - 2gR)$

At position B, we can write the Newton's law as



$N + mg = m\frac{v^2}{R} \Rightarrow N = m\frac{v^2}{R} - mg$

The condition for the sphere not to lose contact at the highest position is $N \geq 0$, so

$\frac{mv^2}{R} - mg \geq 0$

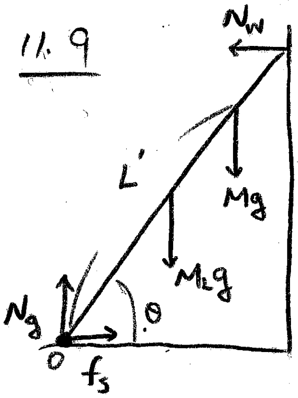
$v^2 \geq gR$

So we have $\frac{10}{7}(gh - 2gR) \geq gR$

$$\frac{10}{7} gh - \frac{20}{7} gR \geq gR$$
$$\frac{10}{7} gh \geq \frac{27}{7} gR$$
$$10h \geq 27R$$
$$h \geq 2.7R$$

$h_{min} = 2.7R$

11.9



As shown in the figure, to prevent slipping, we need $f_s \leq \mu_s N_g$

Force equilibrium in x and y direction indicates that

$$N_g - (M_L + M)g = 0 \quad \text{and} \quad f_s = N_w$$

Torque equilibrium with respect to point O indicates that

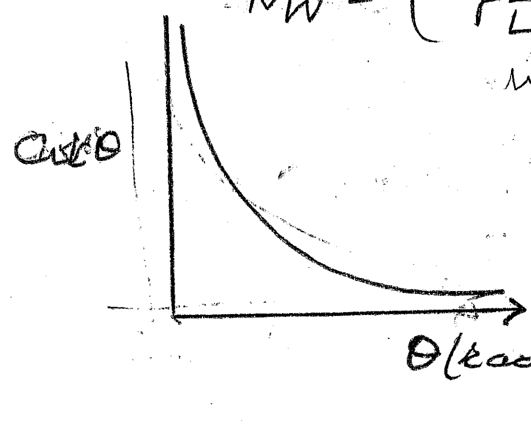
$$(L\sin\theta) N_w - (L'\cos\theta) Mg - (\tfrac{L}{2}\cos\theta) M_L g = 0$$

Combining the equations above, we have

$$f_s = N_w \leq \mu_s N_g = \mu_s (M_L + M) g$$

$$\Rightarrow N_w = \frac{L'}{L}\cot\theta \, Mg + \frac{1}{2}\cot\theta \, M_L g$$

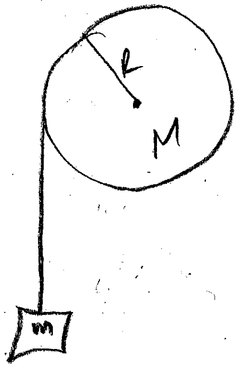
$$N_W = \left(Mg\frac{L'}{L} + \frac{M_L g}{2}\right)\cot\theta$$



when θ becomes very small cotθ becomes very large, N_W will become larger than maximum value of f_s and ladder will slip.

$[f_s \leq \mu_s (M + M_L) g]$

11-10

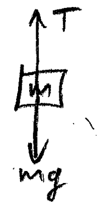


Let's draw the free body diagram for these two objects (mass and pulley) separately.

$$\sum \vec{\tau} = I \vec{\alpha}$$

$$TR\,\hat{z} = \frac{1}{2}MR^2\vec{\alpha} \Rightarrow TR = \frac{1}{2}MR^2\alpha$$

(magnitude only)



$$\sum \vec{F} = m\vec{a}$$

$$mg - T = ma$$

(Let's define positive direction to be downward)

The relation between a and α is $a = R\alpha$. So,

$$mg - T = ma$$
$$mg - \frac{1}{2}MR\alpha = ma$$
$$mg - \frac{1}{2}Ma = ma$$
$$mg = \left(m + \frac{1}{2}M\right)a \Rightarrow a = \frac{mg}{m + \frac{1}{2}M}$$

Note that if $M = 0$ (pulley is massless), m would fall with $a = g$ downward, which is expected.

In 2 seconds, m will drop up to distance

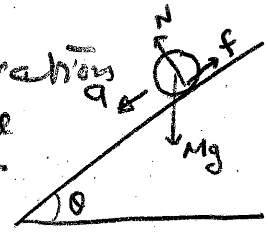
$$s = \frac{1}{2}at^2 = \frac{1}{2}\frac{mg}{m + \frac{1}{2}M}t^2$$

$$= \frac{1}{2}\frac{(1\ \text{kg})(9.8\ \text{m/s}^2)}{1\ \text{kg} + \frac{1}{2}(4\ \text{kg})}(2\ \text{s})^2$$

$$= 6.53\ \text{m}$$

11.11

If no slip, velocity and acceleration at point of contact must be zero at all times.



$$\begin{cases} Mg\sin\theta - f = Ma & \cdots (1) \\ rf = I\alpha & \cdots (2) \\ a = \alpha r & \cdots (3) [\text{No slip}] \end{cases}$$

(3) in (2) $\quad rf = I\dfrac{a}{r} \quad \cdots (4)$

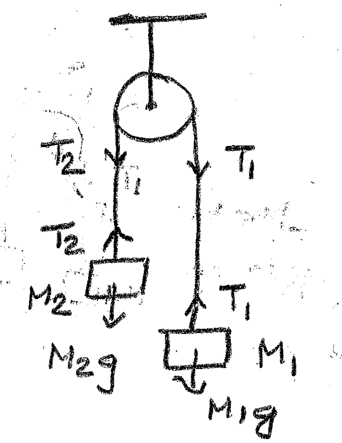
(4) in (1) $\quad Mg\sin\theta - \dfrac{Ia}{r^2} = Ma \Rightarrow a = \dfrac{M}{M+\frac{I}{r^2}} g\sin\theta = \dfrac{1}{1+\frac{I}{Mr^2}} g\sin\theta$

Since $\dfrac{I_{ring}}{Mr^2} > \dfrac{I_{sphere}}{Mr^2}$ in the denominator $\Rightarrow a_{ring} < a_{sphere}$

The sphere will reach first ※

11.12

Now we must Consider the rotation of the pulley, we have $T_1 \neq T_2$, thus



$$\begin{cases} M_1 g - T_1 = M_1 a & \cdots (1) \\ R(T_1 - T_2) = I\alpha = \frac{1}{2} M_p R^2 \left(\frac{a}{R}\right) & \cdots (2) \\ T_2 - M_2 g = M_2 a & \cdots (3) \end{cases}$$

$(1) + \frac{(2)}{R} + (3)$, $(M_1 - M_2)g = (M_1 + \frac{1}{2}M_p + M_2)a \Rightarrow a = \underline{\frac{M_1 - M_2}{M_1 + M_2 + \frac{M_p}{2}} g}$ ※

11.13

$$\Delta P = \frac{\Delta F}{A} = \frac{42N}{\pi\left[(0.25inch)\left(\frac{0.0254m}{1\ inch}\right)\right]^2} = \underline{3.3 \times 10^5 N/m^2 = 33\ N/cm^2}$$ ※

11.14

The net force is $F_{in} - F_{out} = (P_{in} - P_{out})A$

$$= (1atm - 0.1atm)\left(\frac{1.013 \times 10^5 N/m^2}{1atm}\right)(1.4m)^2 = \underline{1.8 \times 10^5 N}$$ ※