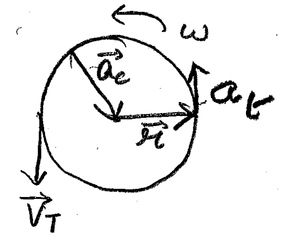
Week (10) Solutions

10-1

$\vec{r}$, $\vec{a}_c$ and $\vec{V}_T$ rotate at ω-radians per second. The angular velocity vector $\vec{\omega}$ is constant and points in a direction perpendicular to the plane of motion.



10-2

$d = 30\,cm \Rightarrow r = 15\,cm$

Assume Counter clockwise rotation

$t = 2\,sec.$

$\vec{\omega}_i = 0$; $\vec{\omega}_f = 33\frac{1}{3}\ rev/min\ \hat{z}$

$$\vec{\omega}_f = 33.33\,\frac{rev}{min}\hat{z} = \frac{33.33\times 2\pi\ rad}{60\,sec}\hat{z} = 3.488\ rad/sec\ \hat{z}$$

i, $\vec{\omega}_f = (\omega_i + \alpha t)\,\hat{z}$

$$\Rightarrow \vec{\alpha} = \frac{\omega_f - \omega_i}{t} = \frac{3.488 - 0}{2} = 1.744\ rad/sec^2\ \hat{z}$$

ii, $\theta = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 = 0 + 0 + \frac{1}{2}\times 1.744\times 25$

$= 21.8\ rad$

$\#$ of Revolutions $= \frac{21.8}{2\pi} \cong 3.5$

iii, $2\times 2\pi = \frac{1}{2}\times 1.744\times t^2$

$$t = \left(\frac{8\times 3.142}{1.74}\right)^{1/2} = 3.8\ secs.$$

iv, $\vec{a}_c = -r\omega^2\hat{r}$; $\vec{a}_\tau = r\alpha\hat{\tau}$

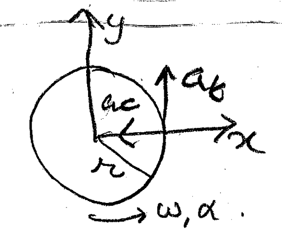
Now at $t = 1\ sec$; $\omega_f = \omega_i + \alpha t$
$= 0 + 1.744\times 1 = 1.744\ rad/sec$

$\therefore\ \vec{a}_c = -0.15\times(1.744)^2 = -0.456\ m/s^2\ \hat{r}$

$\vec{a}_\tau = 0.15\times 1.744 = 0.2616\ m/s^2\ \hat{\tau}$

Radial acceleration is centripetal acceleration

10-3 $\underset{\rightarrow}{\alpha} = \alpha\hat{z}$



$\underset{\rightarrow}{a_c}$ = centripetal acceleration $= -r\omega^2\hat{r}$

and $\underset{\rightarrow}{a_T}$ = tangential acceleration $= r\alpha\hat{\tau}$

$\therefore$ Net acceleration $a = \sqrt{a_c^2 + a_T^2}$

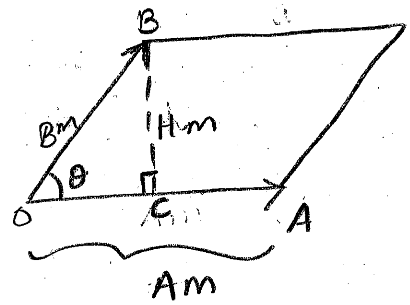
$$= \sqrt{r^2\omega^4 + r^2\alpha^2}$$

$$= r(\omega^4 + \alpha^2)^{1/2}$$

10-4 Area of the parallelogram = base x altitude

$= A \times H$

$= AH\ m^2$.



Now from Δ^{le} OBC; $\sin\theta = \frac{H}{B}$

$\Rightarrow H = B\sin\theta$

$\therefore$ Area $= AH = AB\sin\theta\ m^2$

But $|\underset{\rightarrow}{A} \times \underset{\rightarrow}{B}| = AB\sin\theta\ m^2 = |\underset{\rightarrow}{C}|$

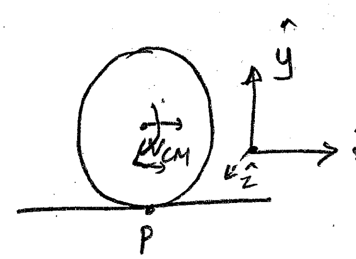
Direction of $\underset{\rightarrow}{C} = \underset{\rightarrow}{A} \times \underset{\rightarrow}{B}$ is perpendicular to the plane and in our diagram; points out of the paper [Right Hand Rule]

10-5 Force causes translation and hence linear acceleration; $M\underset{\rightarrow}{a} = \underset{\rightarrow}{f}$

Torque causes rotation and hence angular acceleration; $I\underset{\rightarrow}{\alpha} = \underset{\rightarrow}{\tau}$.

In order to have a torque, the force must be applied at some distance $\underset{\rightarrow}{r}$ from the axis of rotation and the force must be perpendicular to $\underset{\rightarrow}{r}$.

10-6 $\underset{\rightarrow}{V_{CM}} = 2\frac{m}{s}\hat{x}$; $r = 0.5m$



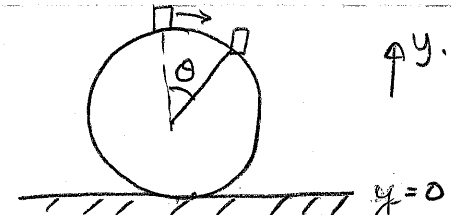
i, velocity $\underset{\rightarrow}{V_P} = 0$ at all times for a perfectly rolling body without slip.

ii, $\underset{\rightarrow}{\omega} = -rV_{CM}\hat{z} = -0.5 \times 2\hat{z} = -1$ rad/sec $\cdot\ \hat{z}$

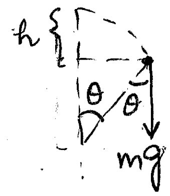
(points into paper)

10-7 $d = 2m \Rightarrow r = 1m$.

$g = 9.8\ m/s^2$



The object will leave the cylinder when normal reaction vanishes.

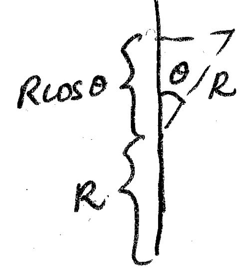


centripetal force at the instant of losing contact is $F_c = mg\cos\theta$.

$\therefore\ mg\cos\theta = \frac{mv^2}{r}$ (velocity at the instant of losing contact $= v$)

$\Rightarrow\ g\cos\theta = \frac{v^2}{r} = v^2$ — ①

Now using conservation of energy.



$PE_f - PE_i = KE_i - KE_f$

$mg(y_f - y_i) = -KE_f$

$mg\left(R(1+\cos\theta) - 2R\right) = -\frac{1}{2}mv^2$

$gR(\cos\theta - 1) = -\frac{1}{2}v^2$

$\Rightarrow\ g(1-\cos\theta) = \frac{v^2}{2}$ — ②

$g - g\cos\theta = \frac{v^2}{2}$

substituting ① in ② ; $g - v^2 = \frac{v^2}{2} \Rightarrow g = \frac{3v^2}{2}$ — ③

substituting ③ back in ① ; $\frac{3v^2}{2}\cos\theta = v^2$

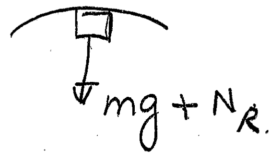
$\Rightarrow \cos\theta = \frac{2}{3}$

$\Rightarrow \theta = 48.2^{\circ}$.

velocity at this point from equation ① is

$g\cdot\cos\theta = v^2 \Rightarrow\ v = \sqrt{9.8 \times \frac{2}{3}} = 2.56\ m/s$.

10-8 $V_T = \sqrt{Rg}$



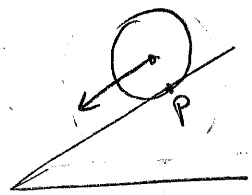
$$\Rightarrow \frac{mv^2}{R} = \frac{m \cdot Rg}{R} = mg$$

$\Rightarrow$ centripetal force is provided by $mg + N_R$

and N_R must be greater than zero

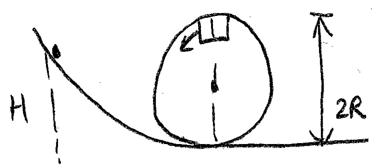
This is a risky situation. We need some normal reaction so that we do not fall off. Hence, v_T has to be slightly greater than $\sqrt{Rg}$.

10-9 Yes, for rolling without slipping, a torque is required which is provided by the friction at the point of contact P.



In pure rolling situation; velocity of point P on wheel $= 0 \Rightarrow$ displacement $= 0 \Rightarrow$ work done by friction $= 0$.

10-10 velocity at top $= \sqrt{Rg}$



By conservation of energy

$$mgH = mg \cdot 2R + \tfrac{1}{2}mv^2$$
$$= mg \cdot 2R + \tfrac{1}{2}m \cdot Rg$$
$$= \frac{5Rgm}{2}$$
$$\Rightarrow H = \frac{5R}{2}$$

This is the minimum height, but the body should be dropped from a slightly higher height to ensure $N \neq 0$ on top.

10-11 $\mu = 0.3$

max friction $= \mu Mg$. This provides acceleration Ma_{max}

$\therefore \mu Mg = Ma_{max} \Rightarrow a_{max} = \mu g = 0.3 \times 9.8$

$= 2.94 \, m/s^2$.