SOLUTIONS - WEEK 1

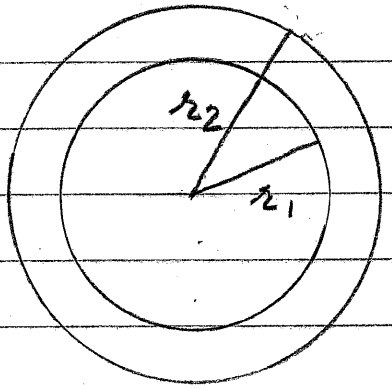
1-1 CIRCUMFERENCE.

$C = 2\pi R$

$C_1 = 2\pi r_1$

$C_2 = 2\pi r_2$

$C_1 - C_2 = 2\pi (r_1 - r_2)$



So if r_1 is 50 m or 100 m and r_2 is 52 m or 102 m the difference is 4π meters.

1-2 The closest two atoms come, distance between their centers is the diameter

$2 \times 0.5 \times 10^{-10}$ m

So # of atoms in 1 cm will be

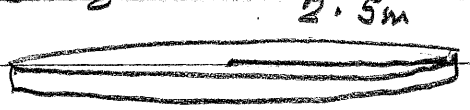
$$= \frac{1\text{ cm}}{10^{-10}\text{m}} = \frac{10^{-2}\text{m}}{10^{-10}\text{m}} = 10^{8}$$

1-3 The volume of a Disk of radius r and thickness t is $V_D = \pi r^2 t$. The slick has a radius of 2.5 m and its thickness is one molecule "size"

$V_D = 0.1\text{ cm}^3$

$= 0.1 \times (10^{-2}\text{m})^3$

$= 10^{-7}\text{m}^3$

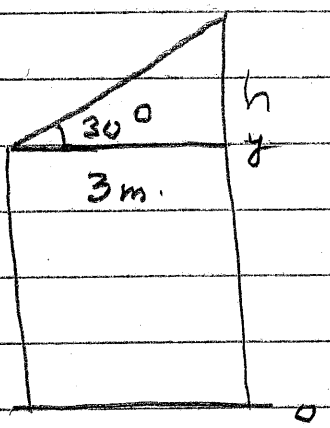


$$t = \frac{10^{-7}}{\pi \times (2.5)^2} = 5.1 \times 10^{-9}\ m$$

1-4 First,

$$\tan\theta = \frac{O}{a}$$

$$= \frac{h}{3}$$



$$h = (3 \tan 30^\circ)\ m.$$

$$y = 6\ ft = (6 \times 0.305)\ m$$

$$= 1.83\ m.$$

$$h = 3 \tan 30^\circ = 1.73\ m.$$

so $y + h = 3.56\ m.$

1-5 $R_E = 6436\ km.$

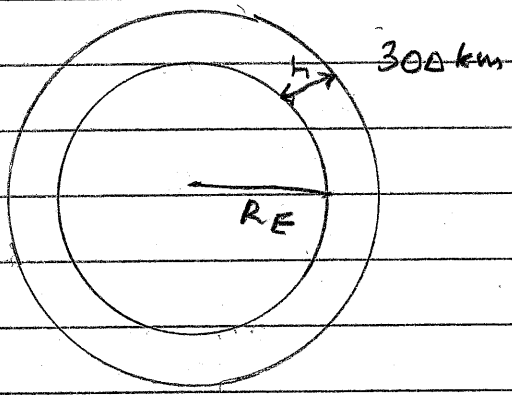
$h = 300\ km$

Radius of orbit

$$R_E + h = 6736\ km$$

$$= 6736 \times 10^3\ m$$

$$= 6.74 \times 10^6\ m.$$

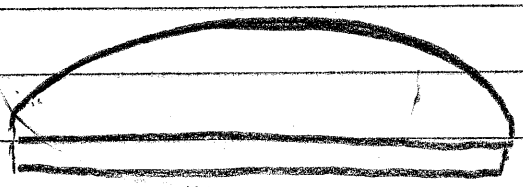


Circumference

$$c = 2\pi (R_E + h)$$

$$= 42.3 \times 10^6\ m$$

1-6 Now we are dealing with half of a disk



$$V = \frac{\pi R^2 L}{2}$$

$R = 2000\,km = 2\times10^6\,m.$

$L = 3000\,m = 3\times10^3\,m.$

$$V = \frac{\pi \times (2\times10^6)^2 \times 3\times10^3}{2}$$

$$= 18.9\times10^{15}\,m^3$$

$1\,m^3 = 10^3$ litres.

$V = 18.9\times10^{18}$ litres.

$1\,m = 39.4''$

$V = 18.9\times10^{15}\times(39.4)^3$ cubic inches

$= 1.15\times10^{21}$ cubic inches.

1-7 $r = (10 \pm 0.2)\,cm.$

Let us define a small distance δr.

$$r = (r' \pm \delta r) = r'\left[1 \pm \frac{\delta r}{r'}\right]$$

$\frac{\delta r}{r'}$ is the uncertainty in r: 2%.

Diameter $2r = 2(r' \pm \delta r)$

$$= 2r'\left[1 \pm \frac{\delta r}{r'}\right]$$

uncertainty 2%

Surface Area $A = 4\pi r^2$

$A = 4\pi (r' \pm \delta r)^2$

$= 4\pi r'^2 \left[1 \pm \frac{2\delta r}{r'} + \left(\frac{\delta r}{r'}\right)^2\right]$

$\left(\frac{\delta r}{r'}\right)^2$ is very small so we neglect it.

uncertainty $\pm \frac{2\delta r}{r'}$ or 4%

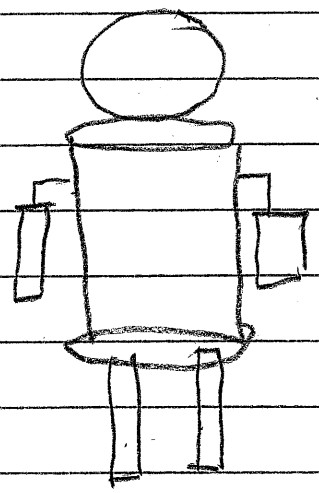
Volume $V = \frac{4\pi}{3} r^3$.

$V = \frac{4\pi}{3} (r' \pm \delta r)^3$

$= \frac{4\pi}{3} r'^3 \left[1 \pm \frac{3\delta r}{r} + \text{very small}\right]$

uncertainty $\pm \frac{3\delta r}{r'}$ or 6%

1-8 Your body is made up of
1 sphere
5 cylinders.



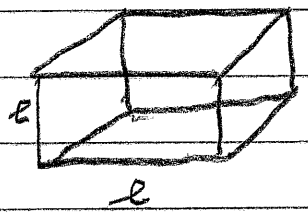
Vol. of sphere = $\frac{4\pi}{3} r^3$

Vol. of cylinder = $\pi r^2 h$.

You do the calculation

→ Immerse in a bath tub, measure the rise in level of water

1-9 A cube has 6 faces



So

Surface area

$= 6\ell^2$

$= 6 \times (1 \times 10^{-6})^2$

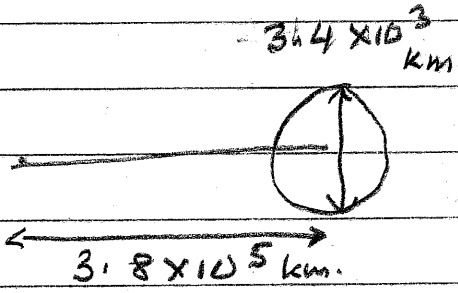
$= 6 \times 10^{-12}\ m^2$

Volume $V = \ell^3 = (10^{-6})^3 = 10^{-18}\ m^3$

1-10 Penny $d = 1.8\ cm$

Nickel $d = 2.1\ cm$

Quarter $d = 2.35\ cm$



Angle subtended by moon

$= \frac{3.4 \times 10^3}{3.8 \times 10^5}$

$= 8.9 \times 10^{-3}$ rad.

To cover it you need for

Penny $D = 2.00\ cm$

Nickel $D = 2.36\ cm$

Quarter $D = 2.64\ cm$

1-11 $b^2 + b^2 = h^2$

$\sin\theta = \frac{o}{h}$ $\cos\theta = \frac{b}{h}$ $\tan\theta = \frac{o}{b}$

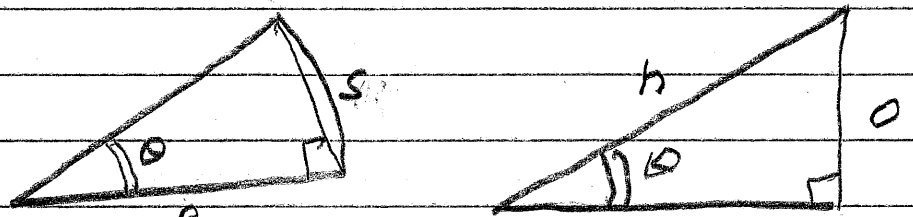
$$\frac{O^2}{h^2} + \frac{b^2}{h^2} = 1$$

$$\sin^2\theta + \cos^2\theta = 1$$

Divide by $\cos^2\theta$

$$\tan^2\theta + 1 = \frac{1}{\cos^2\theta} = \sec^2\theta$$

1-12



$\theta = \frac{S}{R}$ $\qquad$ $\sin\theta = \frac{O}{h}$

If $\theta \ll 1$ the Distinction between chord and arc disappears. So $\sin\theta \to \theta$.

	θ	$\sin\theta$
5°	0.085	0.087
4°	0.068	0.070
3°	0.051	0.052
2°	0.034	0.035
1°	0.017	0.017

$$\cos\theta = (1 - \sin^2\theta)^{1/2}$$

If $\theta \ll 1$ $\qquad \sin\theta = \theta$.

$$\cos\theta = [1 - \theta^2]^{1/2} = 1 - \frac{\theta^2}{2} + \text{very small terms}$$

1-13 $\vec{a} = 5m\,\hat{x}$

$\vec{b} = -3m\,\hat{y}$

$\vec{c} = 3m\,\hat{x} - 6m\,\hat{z}$

$\vec{d} = 1m\,\hat{x} - 2m\,\hat{y} + 3m\,\hat{z}$

1-14 CARDINAL RULE: DIMENSIONS ON LEFT OF EQN MUST EQUAL DIMENSIONS ON RIGHT

$T \rightarrow T^1$

$\ell \rightarrow L^1$

so $T^1 = \sqrt{\frac{L^1}{g}}$ $\qquad g \rightarrow \frac{L^1}{T^2}$

so g must be $L^1 T^{-2}$