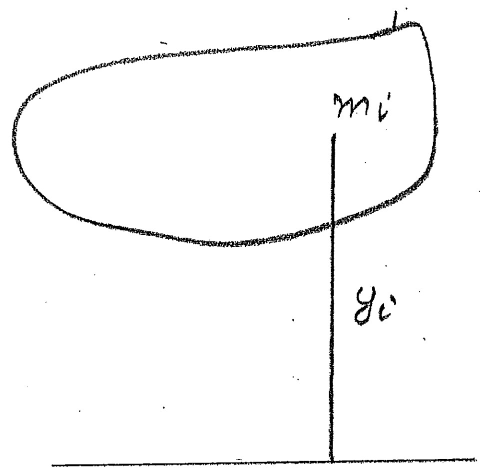
TYPES OF MOTION OF A RIGID BODY

By definition a rigid body consists of many mass points (m_i) located at different points ($\vec{r_i}$) but ($\vec{r_i} - \vec{r_j}$) is fixed so it neither changes shape nor size as it moves. This buys considerable simplification in describing the two types of motion it can have:
For a rigid body one can define a center of gravity and show that it is the same as the center of mass.

$$\vec{r_{cm}} = \frac{\Sigma m_i \vec{r_i}}{\Sigma m_i}$$

Consider a rigid body placed some distance above the Earth.



i) Each mass m_i experiences a force

$$\vec{w_i} = -m_i g\hat{y}$$

Total force on rigid body

$$\vec{w} = \Sigma \vec{w_i} = -\Sigma m_i g\hat{y}$$
$$= -Mg\hat{y}$$

as if it was a single object of mass M.

ii) Each mass m_i has potential energy

$$P_g(i) = m_i g y_i$$

Total potential energy

$$P_g = \Sigma m_i g y_i = Mgy_{cm}$$

Since $y_{cm} = \dfrac{\Sigma m_i g_i}{\Sigma m_i}$

As if it was a single mass M located at the center of mass of the rigid body.

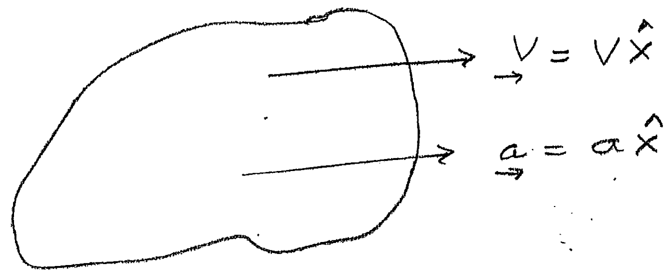
TYPES OF MOTION

Translation: All the masses have the same linear velocity and the same linear acceleration

$$\Sigma \vec{F}_i \neq 0$$

$$\Sigma \vec{\tau}_i = 0$$

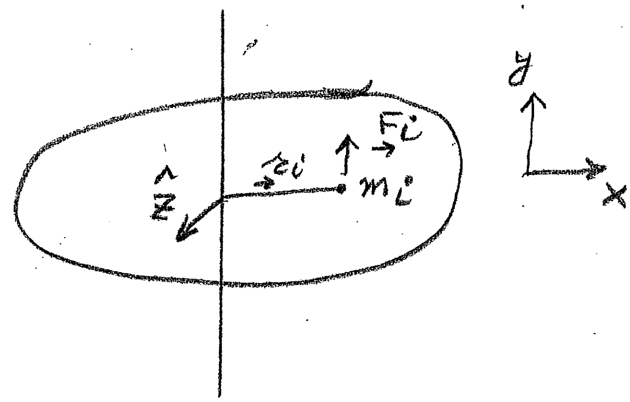
[Indeed $\vec{v} = \vec{v}_{C \cdot G}$]



Rotation about a fixed axis: Now the angular velocity and the angular acceleration are the same for all m_i

Now $\Sigma \vec{F}_i = 0$

$\Sigma \vec{\tau}_i \neq 0$



For equilibrium we need two conditions

$\vec{a} = 0$ and $\vec{\alpha} = 0$ so $\Sigma \vec{F}_i \equiv 0$

$\Sigma \vec{\tau}_i \equiv 0$

All torques taken about a single axis.

The table below summarizes the equations when $\vec{a} \neq 0$ and $\vec{\alpha} \neq 0$. (Dynamics)

Translation (one dimension, x)	Rotation (Fixed axis, Z)
$\vec{x}$	$\vec{\Theta}$
$\vec{v}$	$\vec{\omega}$
$\vec{a} = a\hat{x}$	$\vec{\alpha} = \alpha\hat{z},\ \vec{a}_t = \alpha r_i \hat{\tau}$ *
$\vec{v} = (v_i + at)\hat{x}$	$\vec{\omega} = (\omega_i + \alpha t)\hat{z},\ v_t = \omega r_i \hat{\tau}$ *

$$\underset{\rightarrow}{x} = (x_i + v_i t + \frac{1}{2} a t^2)\hat{x}$$

$$v^2 = v_i^2 + 2a^2(x - x_i)$$

M (Mass)

$$M\underset{\rightarrow}{a} = \Sigma \underset{\longrightarrow}{F_i}$$

At that point
At that time

$$\underset{\rightarrow}{\Theta} = (\Theta_i + \omega_i t + \frac{1}{2} \alpha t^2)\hat{z}$$

$$\omega^2 = \omega_i^2 + 2\alpha^2(\Theta - \Theta_i)$$

Displacement along c

$$\Delta S = r\Delta\Theta$$

$I = \Sigma M_i r_i^2$ (Moment of Inertia)**

$$I\underset{\rightarrow}{\alpha} = \Sigma \underset{\longrightarrow}{\tau_i}$$

About same axis as I

**I measures the manner in which the mass is distributed around the axis so $\Sigma \underset{\longrightarrow}{\tau_i}$ must also be calculated using the same axis.