TYPES OF MOTION OF A RIGID BODY

By definition a rigid body consists of many mass points \((m_i)\) located at different points \((r_i)\) but \((r_i - r_j)\) is fixed so it neither changes shape nor size as it moves. This buys considerable simplification in describing the two types of motion it can have:

For a rigid body one can define a center of gravity and show that it is the same as the center of mass.

\[
\mathbf{r}_{cm} = \frac{\sum m_i r_i}{\sum m_i}
\]

Consider a rigid body placed some distance above the Earth.

\[
i) \quad \text{Each mass } m_i \text{ experiences a force } \mathbf{w}_i = -m_i g \hat{y} \\
\text{Total force on rigid body } \mathbf{w} = \Sigma \mathbf{w}_i = -\Sigma m_i g \hat{y} = -Mg \hat{y}
\]

as if it was a single object of mass \(M\).

\[
ii) \quad \text{Each mass } m_i \text{ has potential energy } \mathcal{P}_z(i) = m_i g y_i \\
\text{Total potential energy } \mathcal{P}_z = \Sigma m_i g y_i = M g y_{cm}
\]

Since \(y_{cm} = \frac{\Sigma m_i g y_i}{\Sigma m_i}\)

As if it was a single mass \(M\) located at the center of mass of the rigid body.
TYPES OF MOTION

Translation: All the masses have the same linear velocity and the same linear acceleration

\[ \sum F_i \neq 0 \]
\[ \sum \tau_i = 0 \]

[Indeed \[ v = v_c \hat{c} \]]

Rotation about a fixed axis: Now the angular velocity and the angular acceleration are the same for all \( m_i \),

Now
\[ \sum F_i = 0 \]
\[ \sum \tau_i = 0 \]

For equilibrium we need two conditions
\[ \boldsymbol{a} = 0 \] and \[ \boldsymbol{\alpha} = 0 \] so
\[ \sum F_i = 0 \]
\[ \sum \tau_i = 0 \]

All torques taken about a single axis.

The table below summarizes the equations when \( \boldsymbol{a} \neq 0 \) and \( \boldsymbol{\alpha} \neq 0 \). (Dynamics)

<table>
<thead>
<tr>
<th>Translation (one dimension, ( x ))</th>
<th>Rotation (Fixed axis, ( Z ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{x} )</td>
<td>( \Theta )</td>
</tr>
<tr>
<td>( v )</td>
<td>( \omega )</td>
</tr>
<tr>
<td>( a = \alpha \dot{x} )</td>
<td>( \alpha = \omega^2 ), ( a_i = \alpha r_i \dot{\hat{r}} \ast )</td>
</tr>
<tr>
<td>( v = (v_i + at) \dot{x} )</td>
<td>( \omega = (\omega_i + at) \dot{\hat{z}} ), ( v_i = \omega r_i \dot{\hat{r}} \ast )</td>
</tr>
</tbody>
</table>
\[ x = (x_i + v_i t + \frac{1}{2} a t^2) \hat{t} \]
\[ v^2 = v_i^2 + 2a^2 (x - x_i) \]

\[ M \text{ (Mass)} \]
\[ M \ddot{a} = \Sigma F_i \]

At that point
At that time

\[ \Theta = (\Theta_i + \omega_i t + \frac{1}{2} \alpha t^2) \hat{\Theta} \]
\[ \omega^2 = \omega_i^2 + 2\alpha^2 (\Theta - \Theta_i) \]

Displacement along \( c \)
\[ \Delta S = r \Delta \Theta \]
\[ I = \Sigma M r_i^2 \text{ (Moment of Inertia)**} \]
\[ I \alpha = \Sigma r_i \]

About same axis as \( I \)

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**I measures the manner in which the mass is distributed around the axis so \( \Sigma r_i \) must also be calculated using the same axis.