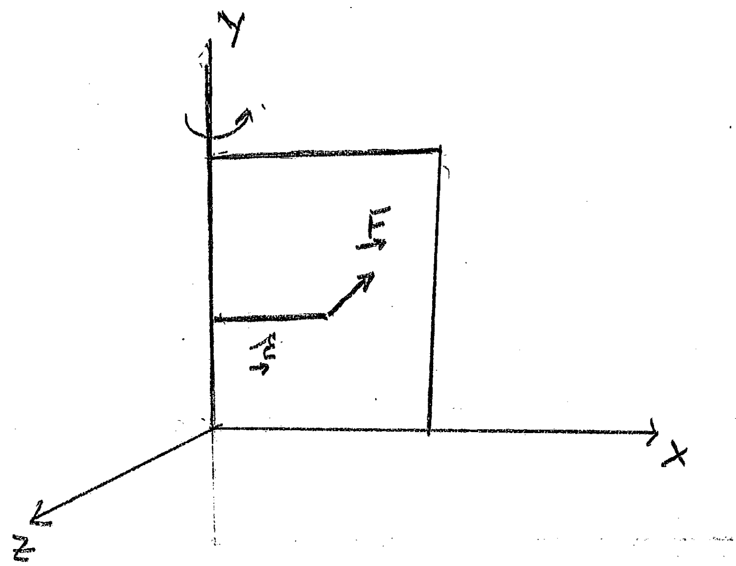
TORQUE

TORQUE: IS THE PHYSICAL AGENCY WHICH IS NECESSARY TO CAUSE ANGULAR ACCELERATION AND HENCE ROTATION ABOUT AN AXIS. WE WILL CONSIDER THE CASE OF ROTATION ABOUT A FIXED AXIS. TO HAVE A TORQUE ONE MUST APPLY A FORCE AT SOME DISTANCE FROM THE AXIS ABOUT WHICH ROTATION IS DESIRED.

Consider the following:
You want to open a door which is hinged along the y-axis.



You pick a point which is some distance $\vec{r}$ from the hinge. Indeed the larger the r the less push (force) you will need to cause the door to swing. Next, you need to apply a force perpendicular to $\vec{r}$. If $\vec{F}$ is parallel to $\vec{r}$ the door will never open. Notice that $\vec{r} \parallel \hat{x}$, $\vec{F} \parallel -\hat{z}$ but door rotates about $\hat{y}$. Indeed the physical agency that causes the swing is the Torque Vector, $\vec{\tau}$ which is parallel to $\hat{y}$. Amazing, $\vec{r}$ is horizontal, $\vec{F}$ is horizontal but $\vec{\tau}$ is vertical.

We need a new concept in vector algebra such that multiplying two vectors produces a third vector which is perpendicular to both of them. Such a product is called a vector product or cross product. Given two vectors $\vec{A}$ and $\vec{B}$ with an angle

$$\Theta = (\vec{A}, \vec{B})$$

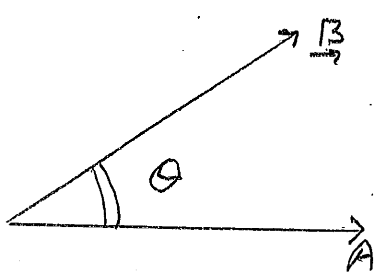
Between them, the vector product is written as

$$\vec{C} = (\vec{A} \times \vec{B})$$

The magnitude of C is

$$C = AB\sin(\vec{A}, \vec{B})$$

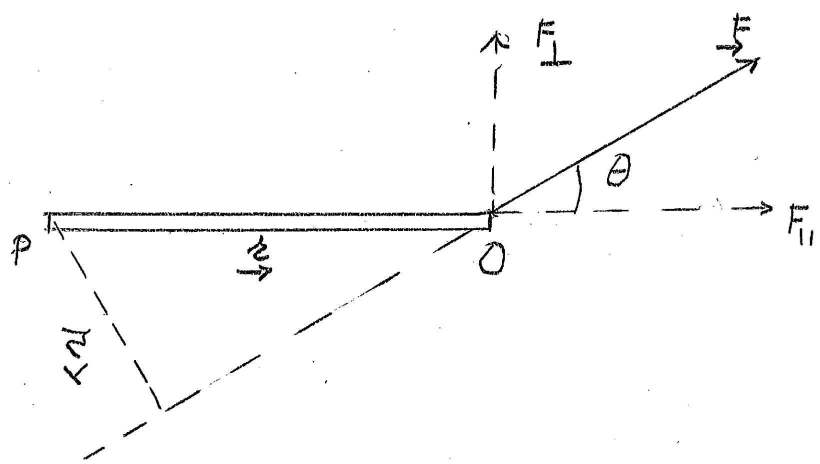
$\vec{C}$ is perpendicular to the AB plane. Which perpendicular?



Right Hand Rule: Stretch right hand

First Vector	$\underset{\rightarrow}{A}$ ||Thumb
Second Vector	$\underset{\rightarrow}{B}$ ||Fingers
Third Vector	$\underset{\rightarrow}{C} \perp$Palm

The Torque Vector can now be defined formally. A bar of length $\underset{\rightarrow}{r}$ can pivot (rotate) about an axis perpendicular to point P. We apply force $\underset{\rightarrow}{F}$ as shown



Torque

$$\underset{\rightarrow}{\tau} = \underset{\rightarrow}{r} \times \underset{\rightarrow}{F}$$

Direction of $\underset{\rightarrow}{r}$ is always from pivot point P to point of application (0) of force $\underset{\rightarrow}{F}$.

Direction of $\underset{\rightarrow}{\tau}$ along $+\hat{z}$

Magnitude of $\tau = rF\sin\Theta = rF_\perp = r_\perp F$

$F_{\perp}$ = Component of $\underrightarrow{F} \perp$ Bar

$r_{\perp}$ = Perpendicular distance between $\underrightarrow{F}$ (extended) and P [sometimes called moment arm].

Immediately one notices

$\underrightarrow{\tau}$ is zero if $\underrightarrow{F} \parallel \underrightarrow{r}$

$\underrightarrow{\tau}$ is maximum when $\underrightarrow{F} \perp \underrightarrow{r}$