SOME CONSEQUENCES OF EARTH'S ROTATION

The Earth is essentially a sphere of radius about 6400km which rotates about its axis once every 24 hours.
So angular Velocity is:

$$w_E = \frac{2\pi}{24 \times 3600} \cong 7.3 \times 10^{-5} rad/s$$

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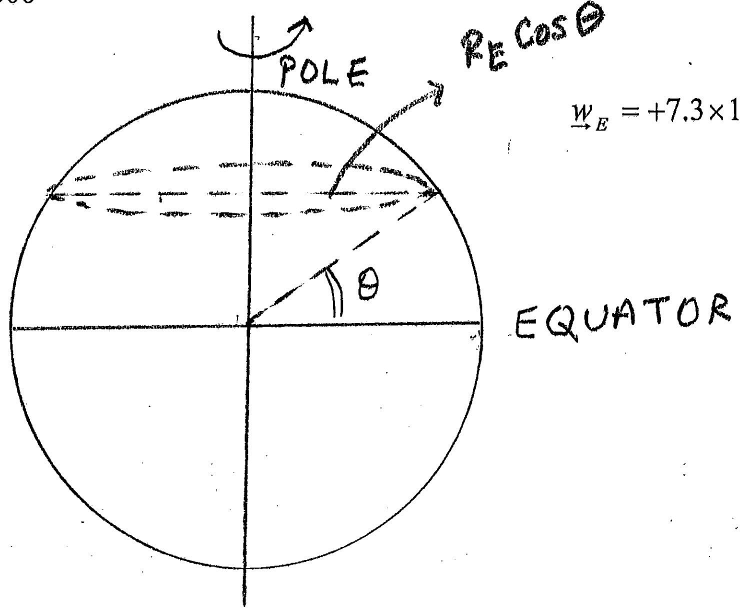


$\underset{\rightarrow}{w}_E = +7.3 \times 10^{-5} rad/s\hat{z}$

So every point on Earth is in uniform circular motion. At latitude Θ the radius of the circle is $R_E Cos\vartheta$ so centripetal acceleration is

$$\underset{\rightarrow}{a_c} = -R_E \cos\Theta w_E^2 \hat{r}$$

At the pole $\Theta = \pi/2$, $\underset{\rightarrow}{a_c} = 0$

At the equator $\Theta = 0$, $\underset{\rightarrow}{a_c} = -R_E w_E^2 \hat{r} \cong -0.03 m/s^2 \hat{r}$

So CONSEQUENCE I:

Our assumption that systems fixed with respect to Earth's surface are Inertial is not precisely correct; except at the Poles. The error is small because $g = 9.8m/s^2$ but it is important.

CONSEQUENCE II:

The apparent weight is not the same at all Θ. At the pole and at the equator the answer is simple because $\underset{\rightarrow}{a}_c \parallel \underset{\rightarrow}{g}$ (both along radius of the Earth)

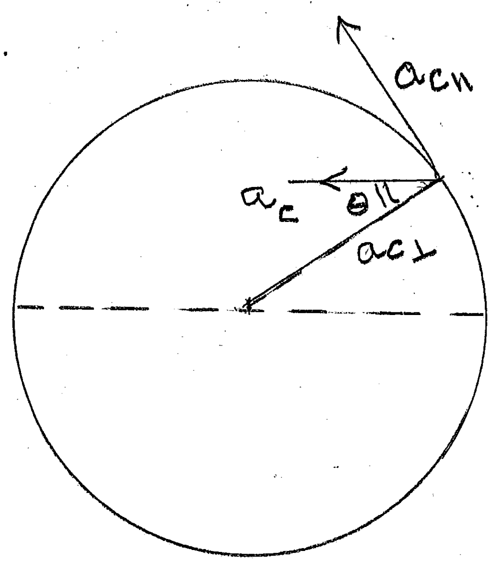
At the pole $\underset{\rightarrow}{a_c} = 0$ $N_R - M_g = 0$ $N_R = M_g$

At the Equator
$(N_R - M_g)\hat{r} = -R_E w_E^2 \hat{r}$

So $N_R = M(g - R_E w_E^2)\hat{r}$
$= M(9.8 - 0.03) m/s^2 \hat{r}$
weight is reduced by about 0.3%

CONSEQUENCE III:

This Is the most subtle and happens for any Θ other than 0 and $\frac{\pi}{2}$ because $\underset{\rightarrow}{a_c}$ is NOT parallel to $\underset{\rightarrow}{g}$.



$\underset{\rightarrow}{a_c} = R_E w_E^2 \cos\Theta$

Indeed now $\underset{\rightarrow}{a}_c$ has a component parallel to surface of Earth

$$a_{c\|} = R_E w_E^2 \sin\Theta\cos\Theta$$

and a component along radius of Earth

$$a_{c\perp} = -R_E w_E^2 \cos^2\Theta\hat{r}$$

which is along r so it modifies "g" slightly.
Since we have an $a_{c\|}$, if you try to hang a pendulum, it cannot be vertical (parallel to $\hat{r}$).
It must tilt to yield a force to produce $a_{c\|}$

$$\tan\delta = \frac{a_{c\parallel}}{g}$$

$$\cong \frac{\sin\Theta\cos\Theta R_E w_E^2}{g}$$

$\delta \to 0$ $\Theta = 0$ and $\Theta = \pi/2$

The situation is exactly like the case of a pendulum hanging in a cart which has an acceleration $\underset{\to}{a} = -a\hat{x}$. (Prob. 4-16)

$$-T\sin\Theta = -Ma$$
$$T\cos\Theta - Mg = 0$$

$$\tan\Theta = \frac{a}{g}$$