PRESSURE OF A GAS – KINETIC ENERGY

Question: Why does a gas exert pressure on the walls of its container?

Answer: At a finite temperature the atoms of the gas are all in random motion. Each time an atom of mass \( m \) and velocity \( +u\hat{x} \) for example has an elastic collision with the wall, it delivers an impulse

\[ +2mu\hat{x} \]

to the wall. If we calculate the number of collisions per second \( (n_s) \), \( (n_s \times 2mu) \) is the change in the momentum of the wall per second, which is a

FORCE

If you divide the force by the area of the wall on which collisions occur you have

\[ \text{pressure} = \frac{-\text{Force}}{-\text{Area}} \]

Proof: Consider a gas which has \( N \) atoms of mass \( m \) in a container of volume \( V \).

Number density \( n = \frac{N}{V} \)

The atoms are in random motion that means they have a distribution of velocities

<table>
<thead>
<tr>
<th># per ( m^3 )</th>
<th>Velocity</th>
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</thead>
<tbody>
<tr>
<td>( n_1 )</td>
<td>( c_1 )</td>
</tr>
<tr>
<td>( n_2 )</td>
<td>( c_2 )</td>
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<td>( \ldots )</td>
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<td>( \ldots )</td>
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<tr>
<td>( n_i )</td>
<td>( c_i )</td>
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</table>

\[ \Sigma n_i = n \]

Since motion is totally random average velocity must be ZERO!

\[ <\vec{c}> = \frac{\Sigma c_i n_i}{\Sigma n_i} \]

Let us consider the \( n_i \) atoms whose velocity is \( c_i \) and focus on the x-component of their velocity \( u_i \) and have it collide with a vertical wall.
As discussed previously. Since mass of wall is enormous compared to atom, wall can pick up momentum but NO kinetic energy so

\[ \Delta p_{wall} = 2mu_i \Delta \mathbf{r} \]

Construct a parallel-piped area \( A \) and height \( u_i \Delta t \).

It is clear that all \( i \) type particles travelling to the right (hence \( \frac{n_i}{2} \) since motion is random) will hit the wall at time \( \Delta t \)

\[ \text{# of collisions in time } \Delta t = \frac{n_i}{2} u_i \Delta t A \]

\[ \text{# of collisions per sec} = \frac{n_i u_i A}{2} \]

\[ \text{Mom}^* \text{ delivered to wall per sec} = \frac{2mu_i n_i u_i A}{2} = m n_i u_i A \]

That is the force on the wall due to type \( i \) particles

\[ F_i = mn_i u_i^2 A \]

Pressure due to them is

\[ P_i = \frac{F_i}{A} = mn_i u_i^2 \]

Pressure due to all \( n \) atoms

\[ P = \Sigma P_i = \Sigma mn_i u_i^2 \]

\[ = nm < u^2 > \]

Since average of \( u^2 \) is

\[ < u^2 >= \frac{\Sigma n_i u_i^2}{n} \]

Since motion is random if \( u, v, w \) are the components of velocity along \( x, y, z \)

\[ < u^2 > = < v^2 > = < w^2 > = \frac{< c^2 >}{3} \]

Because

\[ < u^2 > + < v^2 > + < w^2 > = < c^2 > \]
So pressure

\[ P = \frac{n}{3} \langle m \rangle < c^2 > \]

Of course average kinetic energy of an atom is \( K = \frac{1}{2} m \langle c^2 \rangle \)

So

\[ P = \frac{2}{3} \frac{K \cdot E}{vol} \quad \text{(Pressure is } \frac{2}{3} \text{ of kinetic energy per unit vol)} \]

Also

\[ PV = N k_B T \quad \text{from expt.} \quad n = \frac{N}{V} \]

\[ P = N k_B T = \frac{1}{3} mn < c^2 > \]

Hence

\[ \frac{1}{2} m \langle c^2 \rangle = \frac{2}{3} k_B T \]

Kinetic energy stored in random motion of the atoms

We define the root mean square speed

\[ v_{rms} = \sqrt{\langle c^2 \rangle} = \sqrt{\frac{3 k_B T}{m}} \]

For example: \textit{He} atoms \( m = 4 \times 1.6 \times 10^{-27} \text{ kg} \) so at room temperature \( T = 300 \text{K} \)

\[ v_{rms} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{4 \times 1.6 \times 10^{-27}}} = 1.4 \times 10^3 \text{ m/s} \]

\textit{He} \quad Kr \\ Mixture \quad \text{at 300K} \\
\text{same} \quad \text{Kinetic} \quad \text{Energy}

\textit{Kr} atoms

\( m = 84 \times 1.6 \times 10^{-27} \text{ kg} \)

\[ v_{rms} = \frac{1.4 \times 10^3}{\sqrt{21}} \text{ m/s} \]

\( \approx 3 \times 10^2 \text{ m/s} \)