PRESSURE OF A GAS – KINETIC ENERGY

Question: Why does a gas exert pressure on the walls of its container?

Answer: At a finite temperature the atoms of the gas are all in random motion. Each time an atom of mass m and velocity $+u\hat{x}$ for example has an elastic collision with the wall, it delivers an impulse

$$+2mu\hat{x}$$

to the wall. If we calculate the number of collisions per second (n_s), $(n_s \times 2mu)$ is the change in the momentum of the wall per second, which is a

FORCE

If you divide the force by the area of the wall on which collisions occur you have

$$pressure = \frac{Force}{Area}$$

Proof: Consider a gas which has N atoms of mass m in a container of volume V.

Number density $\qquad n = \frac{N}{V}$

The atoms are in random motion that means they have a distribution of velocities

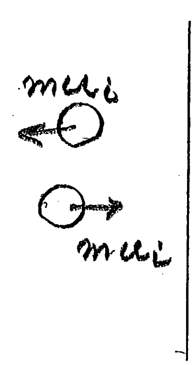
# per m^3	Velocity
n_1	$\underset{\rightarrow}{c_1}$
n_2	$\underset{\rightarrow}{c_2}$
.	.
.	.
.	.
.	.
n_i	$\underset{\rightarrow}{c_i}$

$\Sigma n_i = n$

Since motion is totally random average velocity must be ZERO!

$$< \underset{\rightarrow}{c} > = \frac{\Sigma \underset{\rightarrow}{c_i} n_i}{\Sigma n_i}$$

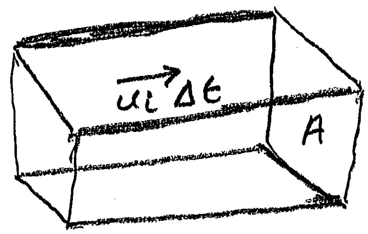
Let us consider the n_i atoms whose velocity is $\underset{\rightarrow}{c_i}$ and focus on the x-component of their velocity u_i and have it collide with a vertical wall.



As discussed previously. Since mass of wall is enormous compared to atom, wall can pick up momentum but NO kinetic energy so

$$\Delta \underrightarrow{p_{wall}} = +2mu_i\hat{x}$$

Construct a parallel-piped area A and height $u_i\Delta t$.



It is clear that all i type particles travelling to the right (hence $\frac{n_i}{2}$ since motion is random) will hit the wall at time Δt

$$\#\text{ of collisions in time } \Delta t = \frac{n_i}{2} u_i \Delta t A$$

$$\#\text{ of collisions per sec} = \frac{n_i u_i A}{2}$$

$$Mom^m \text{ delivered to wall per sec} = \frac{2mu_i n_i u_i A}{2}$$
$$= mu_i n_i u_i A$$

That is the force on the wall due to type i particles

$$F_i = mn_i u_i^2 A$$

Pressure due to them is

$$P_i = \frac{F_i}{A} = mn_i u_i^2$$

Pressure due to all n atoms

$$P = \Sigma P_i = \Sigma mn_i u_i^2$$
$$= nm < u^2 >$$

Since average of u^2 is

$$< u^2 >= \frac{\Sigma n_i u_i^2}{n}$$

Since motion is random if u, v, w are the components of velocity along x, y, z

$$< u^2 >=< v^2 >=< w^2 >= \frac{< c^2 >}{3}$$

Because $$< u^2 > + < v^2 > + < w^2 >=< c^2 >$$

So pressure

$$P = \frac{1}{3} mn < c^2 >$$

Of course average kinetic energy of an atom is $K = \frac{1}{2} m < c^2 >$

So

$$P = \frac{2}{3} \frac{K \cdot E}{vol} \quad \text{(Pressure is } \frac{2}{3} \text{ of kinetic energy per unit vol)}$$

Also $\quad PV = Nk_B T \quad$ from expt. $\quad n = \frac{N}{V}$

$$P = Nk_B T = \frac{1}{3} mn < c^2 >$$

Hence $\quad \frac{1}{2} m < c^2 > = \frac{2}{3} k_B T$

Kinetic energy stored in random motion of the atoms

We define the root mean square speed

$$v_{rms} = \sqrt{< c^2 >} = \sqrt{\frac{3k_B T}{m}}$$

For example: *He* atoms $m = 4 \times 1.6 \times 10^{-27}\, kg$ so at room temperature $T = 300$K

$$v_{rms} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{4 \times 1.6 \times 10^{-27}}} == 1.4 \times 10^3\, m/s$$

He *Kr* *Mixture* *at 300K*

same

Kinetic

Energy

Kr atoms

$m = 84 \times 1.6 \times 10^{-27}\, kg$

$$v_{rms} = \frac{1.4 \times 10^3}{\sqrt{21}}\, m/s$$

$$\approx 3 \times 10^2\, m/s$$