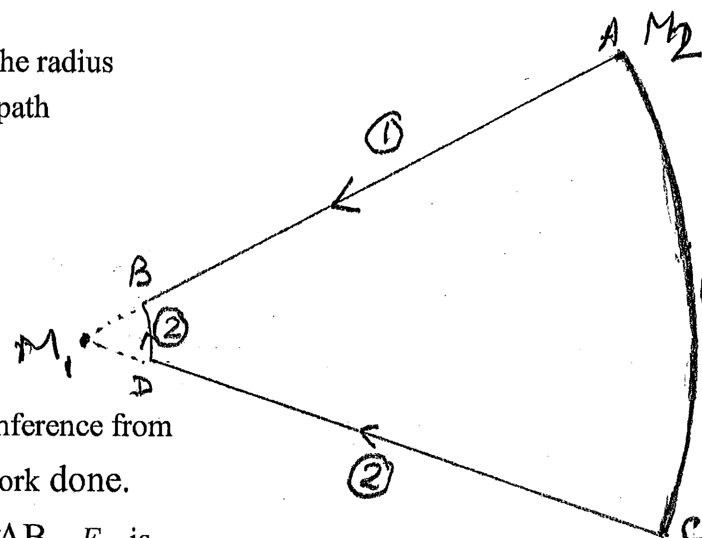
POTENTIAL ENERGY-GRAVITATIONAL FORCE

We can define a potential energy for the Gravitation force. We begin by proving that

$$\underset{\longrightarrow}{F_G} = \frac{-GM_1M_2}{r^2}\hat{r}$$

is a conservative force. That is work done is independent of the path. The crucial point here is that $\underset{\longrightarrow}{F_G}$ is directed along the line joining $\underline{M_1}$ and $\underline{M_2}$. We will use this to prove that $\underset{\longrightarrow}{F_G}$ is conservative. Let us fix M_1 at $r = 0$ and move M_2.

Path 1
We take M_2 from A (R_A) along the radius to point B (R_B), $\underset{\longrightarrow}{F_G}$ is parallel to path we can calculate ΔW_{AB}



Path 2
Start at A and go along the circumference from A to C. Now $\underset{\longrightarrow}{F_G}$ ⊥ path so no work done.
Now go along Radius CD = AB. $\underset{\longrightarrow}{F_G}$ is same, displacement is same so $\Delta W_{CD} = \Delta W_{AB}$

Now go along circumference DB. Again

$$\Delta W_{DB} = 0 \ [\underset{\longrightarrow}{F_G} \perp \text{Displacement}]$$

So

$$\Delta W_{AB} = \Delta W_{ABCD}$$

WORK DONE IS INDEED INDEPENDENT OF PATH!

Applications

Case I Potential Energy of M_1, M_2 system (two point masses)
Place M_1 at $r = 0$

Force on M_2 is $\underset{\longrightarrow}{F_G} = \frac{-GM_1M_2}{r^2}\hat{r}$

Case III Solid uniform sphere centered at $r = 0$ and m at r

Now $\underset{\rightarrow}{F_G} = \frac{-GMm}{r^2} \qquad r > R$

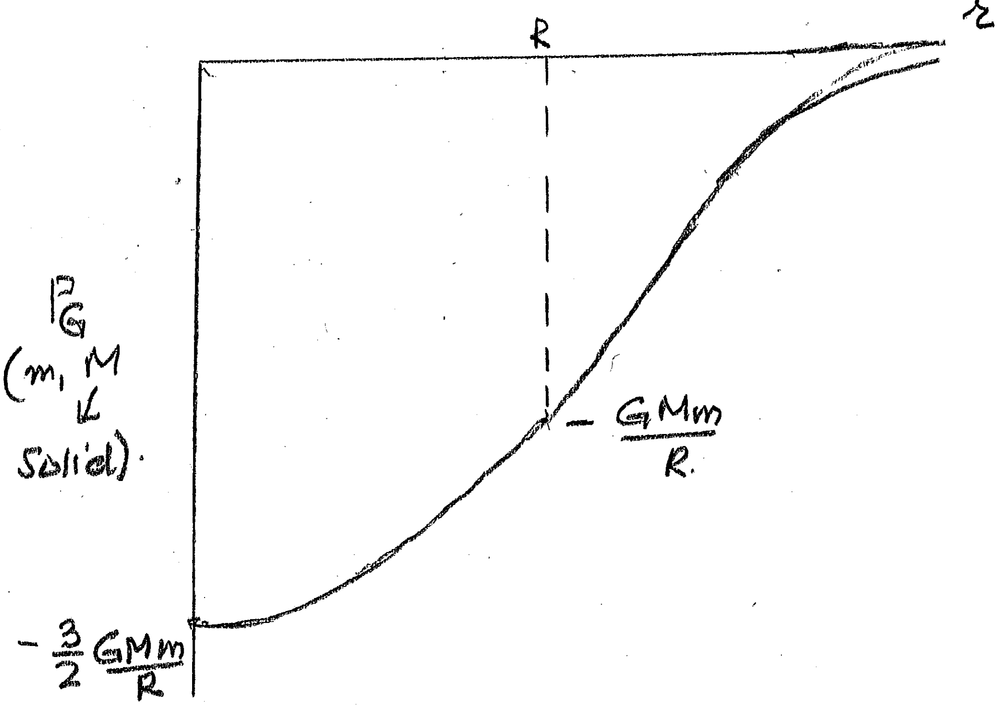
$\underset{\rightarrow}{F_G} = -\frac{4\pi}{3} Gdmr \qquad r < R$

We get

$r > R \qquad P_G = -\frac{GMm}{r}$

$r = R \qquad P_G = -\frac{GMm}{R}$

$r < R \qquad P_G = -\frac{GMm}{R} - \frac{GMm}{2R}\left(1 - \frac{r^2}{R^2}\right)$



Case IV Special case m just outside Earth at height h. Then $r = R_E + h$, $h << R_E$

Potential Energy of Earth – Mass System

$$P_G(h) = -\frac{GM_E m}{R_E + h} = -\frac{GM_E m}{R_E\left(1 + \frac{h}{R_E}\right)} = -\frac{GM_E m}{R_E}\left(1 + \frac{h}{R_E}\right)^{-1}$$

Since $\frac{h}{R_E} << 1$ $\qquad \left(1+\frac{h}{R_E}\right)^{-1} = 1-\frac{h}{R_E}$

$$P_G(h) = -\frac{GM_E m}{R_E}\left(1-\frac{h}{R_E}\right) = -\frac{GM_E m}{R_E} + \frac{GM_E m}{R_E^{\ 2}} h$$

$$P_G(h) = -\frac{GM_E m}{R_E} + mgh$$

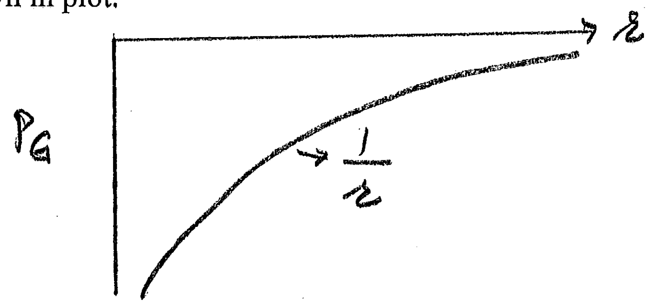
Recall that we previously wrote *mgh* for the Earth Mass system. Note that in fact our potential energy is very large and NEGATIVE. That ensures that we stay close to the Earth.

Change of Potential Energy $\quad\quad \Delta P = -\Sigma \underrightarrow{F_G} \bullet \underrightarrow{\Delta r}$

To calculate P when M_2 is at r we must calculate work needed to put M_2 at r starting from some point where P is zero. Since $\underrightarrow{F_G} \to 0$ as $r \to \infty$ we choose P to equal zero when M_2 is very far away and calculate work done to bring M_2 to r. We will get

$$P_G(M_1, M_2) = \frac{-GM_1M_2}{r}$$

P_G is negative everywhere as shown in plot.



<u>Case II</u> Mass shell centered at $r = 0$, and point mass m at r.

Now $\quad \underrightarrow{F_G} = \frac{-GMm}{r^2} \quad\quad r > R$

$\quad\quad \underrightarrow{F_G} = 0 \quad\quad r < R$

We get

$$P_G = -\frac{GMm}{r} \quad\quad r > R$$

$$P_G = -\frac{GMm}{R} \quad\quad r < R$$