NEWTON'S LAWS (POINT OBJECTS)

FIRST

OBJECTS DO NOT CHANGE THEIR STATE OF MOTION (vel. $\vec{v}$ For Now) SPONTANEOUSLY

DEFINES INERTIA

Examples: SEAT BELTS, STUMBLE, GRASS MOWING

SECOND

a) EVERY OBJECT HAS AN INTRINSIC PROPERTY CALLED INERTIAL MASS (M)

b) AN OBJECT OF MASS M CAN HAVE A NON-ZERO ACCELERATION IF AND ONLY IF THERE IS A FORCE $\vec{F}$ PRESENT SUCH THAT

$$\boxed{M\vec{a} = \vec{F}}$$

COROLLARIES: (i) IF AN OBJECT IS IN EQUILIBRIUM ($\equiv m$) ($\vec{a} = 0$), THE VECTOR SUM OF ALL THE FORCES ACTING ON IT MUST BE ZERO

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 \ldots = 0$$

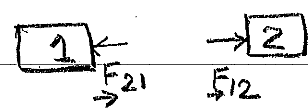
Object does not have to be at rest, it must not change $\vec{v}$.

(ii) IF $\vec{a} \neq 0$ at a SPACE POINT AT A TIME t, THERE MUST BE A FORCE ACTING AT THE PT AT THAT TIME.

THIRD

WHEN TWO OBJECTS INTERACT THE FORCES ACTING ON THEM FORM ACTION-REACTION PAIRS ($\vec{F}_{21}$ acts on object 1, $\vec{F}_{12}$ on object 2)

$$\vec{F}_{21} = -\vec{F}_{12}$$



FORCES

In order to use Newton's Laws we need the Forces that occur in various physical systems. For our discussion in 121 we deal with Mechanical Forces only. Also, we do not discuss in detail the origin of the force in Every Case.

I WEIGHT Near Earth Every unsupported object has an acceleration

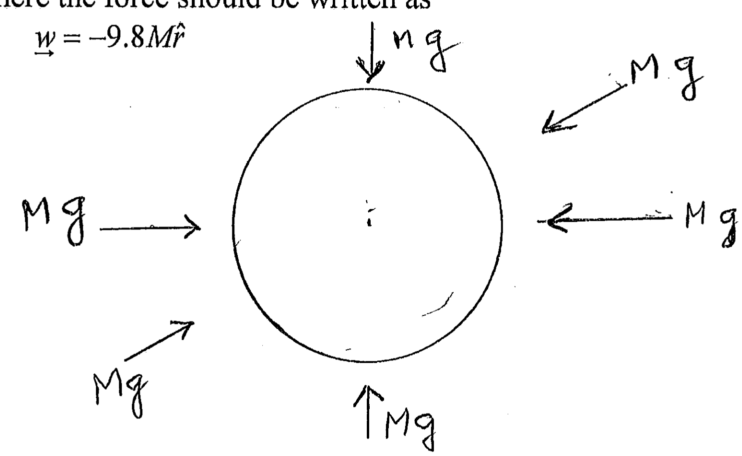
$$\vec{a} = -9.8 m/s^2 \hat{y} = -g\hat{y}$$

So it must experience a force

$$\vec{w} = -9.8 M\hat{y} = -Mg\hat{y}$$

where M is its mass. This force is weight and is a vector perpendicular to the Earth's surface directed toward the center of the Earth. More precisely, since the Earth is a sphere the force should be written as

$$\vec{w} = -9.8 M\hat{r}$$



where $\hat{r}$ is a unit vector along the radius. It is a manifestation of Newton's universal law of Gravitation (discussed in detail later)

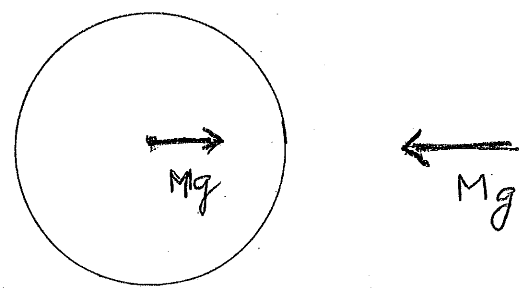
$$\underset{\rightarrow}{F_G} = \frac{-GM_E M}{R_E^{\;2}}\hat{r}$$

where M_E = Mass of Earth

R_E = Radius of Earth

$$G = 6.7 \times 10^{-11} \frac{N-m^2}{(kg)^2}$$

By Newton's 3rd law it follows that the reaction force to $\underset{\rightarrow}{w}$ acts at the center of the Earth



So Earth pulls on M, M pulls on Earth with an Equal and opposite force.

II CONTACT FORCE OR NORMAL FORCE

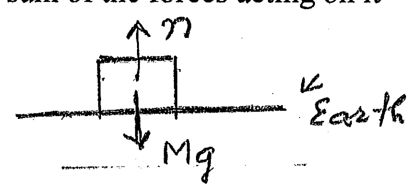
Comes into play when an object is in contact with the surface of a solid. It acts perpendicular to surface of the solid: hence Normal Force (N_R). It comes about because the atoms/molecules of a solid oppose the attempt by any foreign object to enter the solid. For example, put the mass M of the above discussion on the Earth. Now M is in $\equiv m$ so the sum of the forces acting on it must be zero.

$\underset{\rightarrow}{n} = n\hat{y}$

$\underset{\rightarrow}{w} = -Mg\hat{y}$

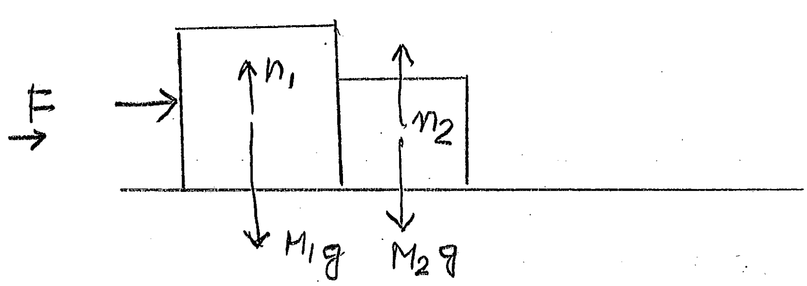
$\underset{\rightarrow}{n} + \underset{\rightarrow}{w} = 0$

$\underset{\rightarrow}{n} = +Mg\hat{y}$



Ex 2 M_1 and M_2 are lying on a smooth horizontal surface. Apply a force $\underset{\rightarrow}{F} = F\hat{x}$ to M_1 as shown. Both M_1 and M_2 acquire an acceleration

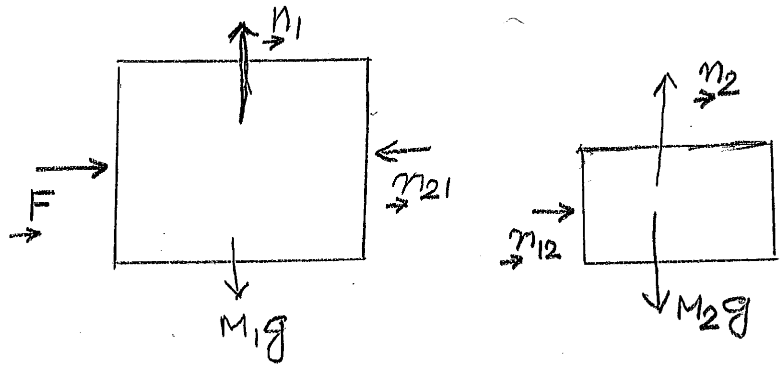
$$\underset{\rightarrow}{a} = a\hat{x}$$



Question: Which force causes M_2 to accelerate?

Answer: Contact force between M_1 and M_2.

Let us draw all the forces acting on each mass (Free Body diagrams)



By the 3rd law $\underset{\to}{n_{12}} + \underset{\to}{n_{21}} = 0$

$$(n_{12} + n_{21})\hat{x} = 0$$

To calculate $\underset{\to}{a}$ we must use force acting at that mass at that time so

$$M_1\underset{\to}{a} = \underset{\to}{F} + \underset{\to}{n_{21}} = F\hat{x} - n_{21}\hat{x}$$

$$M_2\underset{\to}{a} = \underset{\to}{n_{12}} = n_{12}\hat{x}$$

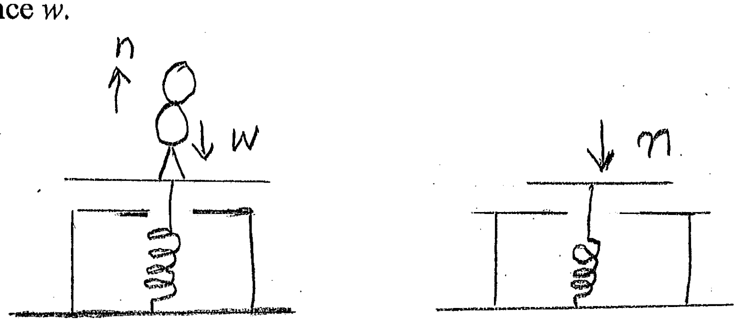
Add

$$(M_1 + M_2)\underset{\to}{a} = F\hat{x} + (n_{21} - n_{12})\hat{x}$$

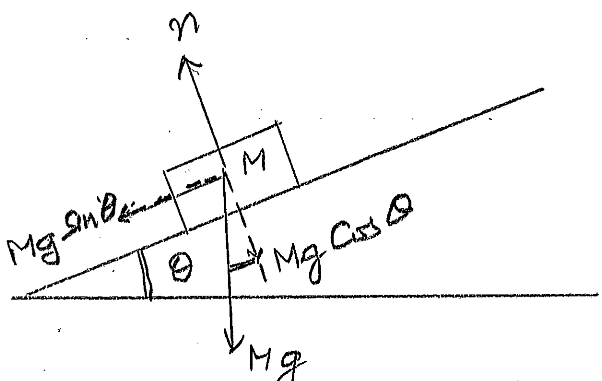
$$= F\hat{x}$$

$$\underset{\to}{a} = \left(\frac{F}{M_1 + M_2}\right)\hat{x}$$

EX 3 To weigh yourself you stand on a weighing machine. You have two forces acting on you $\underset{\to}{w} = -Mg\hat{y}$ and $\underset{\to}{n} = n\hat{y}$ the normal force which the machine exerts on you. You are in $\equiv m$ so $n = Mg$. You push down on machine with $\underset{\to}{n} = -n\hat{y}$ so machine records n and hence w.



EX 4 If the surface is not horizontal $\underset{\to}{n}$ will have to adjust so that three is $\equiv m$ perpendicular (normal) to the surface while there is acceleration $g \sin \Theta$ down the ramp. The force picture is



$\perp$ to surface there is $\equiv m$ so

$$n - Mg\cos\Theta = 0; \quad n = Mg\cos\Theta$$

$\parallel$ to surface there is acceleration caused by $Mg\sin\Theta$

Not surprisingly $\underset{\rightarrow}{n}$ is maximum when $\Theta = 0$ (horizontal surface) and goes to zero when $\Theta = \frac{\pi}{2}$ (surface is vertical)

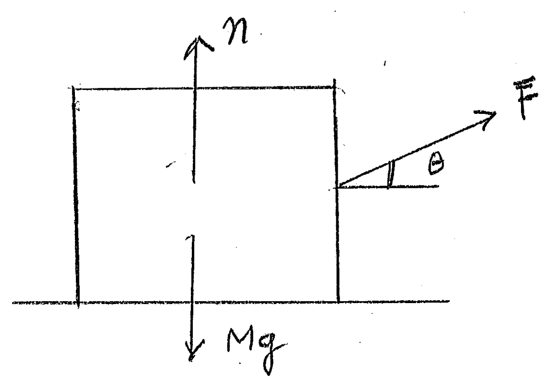
<u>EX 5</u> Another way to change n is to apply a force $\underset{\rightarrow}{F}$ at an angle Θ above the x-axis. Now for $\equiv m$ along y we have

$$n + F\sin\Theta - Mg = 0$$

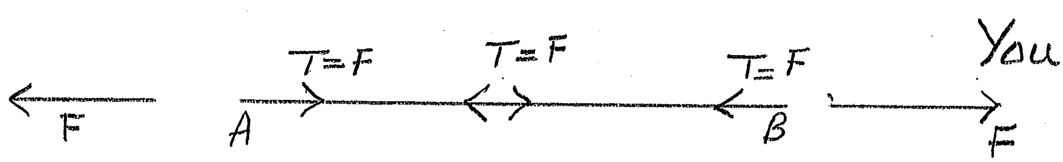
so $$n = Mg - F\sin\Theta$$

while along x there is acceleration given by

$$M\underset{\rightarrow}{a} = F\cos\Theta\hat{x}$$



III <u>TENSION IN A MASSLESS, INEXTENSIBLE STRING</u>



You are holding one end of a light string. Your friend catches hold of the other end. Suppose she pulls on it with a force $\underset{\rightarrow}{F} = -F\hat{x}$, toward the left. In order to keep it in $\equiv m$ you have to pull on the right with $\underset{\rightarrow}{F} = +F\hat{x}$. How come? Well, when she applied $-F\hat{x}$ at A and the string wants to be in $\equiv m$ it must develop $+F\hat{x}$ at A, gain a to keep $\equiv m$ everywhere inside, it needs $\underset{\rightarrow}{F}$'s at every point balancing each other out until point B is reached where string pulls to the left. So for $\equiv m$ at B you must pull with $+F\hat{x}$. A force $\underset{\rightarrow}{F}$ applied at one end of the string causes a tension

$T = F$ to appear in the string such that at the ends $\underset{\rightarrow}{T}$ acts toward the middle and at the middle $\underset{\rightarrow}{T}$ is directed toward the ends.

IV SPRING FORCE (HOOKE'S LAW)

We already used this in establishing a measure for Force because we used a SPRING BALANCE. This force appears if you stretch a spring or squeeze it. The spring resists the change in its length so this force always opposed the stretch (or squeeze). For small changes in length the force is proportional to the change in length hence we write

$$\underset{\rightarrow}{F_{SP}} = -k(\Delta x)\hat{x}$$

where Δx = change in length
k = spring constant

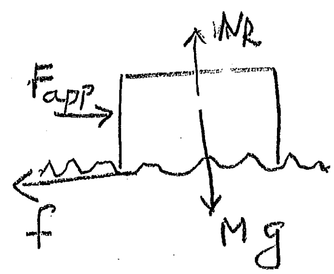
Minus sign ensures that $\underset{\rightarrow}{F_{SP}}$ is opposite to $\underset{\rightarrow}{\Delta x} = \Delta x\hat{x}$.

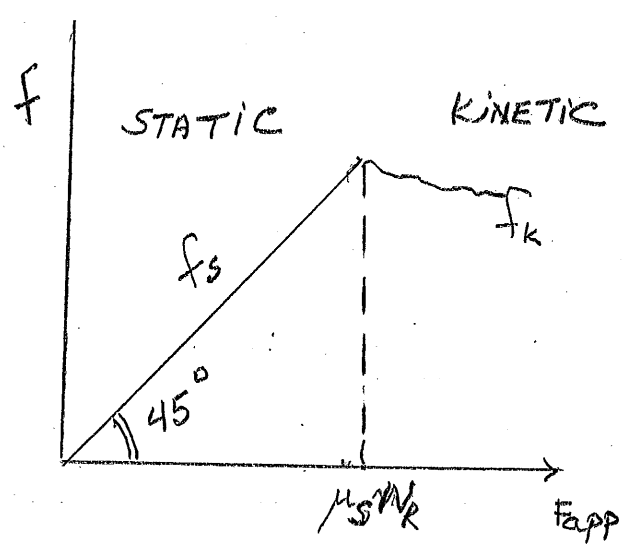
So if $k = 10^4\, N/m$, it will cost you $10N$ of force to change its length by $1mm$.

V FRICTION

This force arises because surfaces of solids are never totally smooth so when two surfaces are made to slide past one another they resist it by developing the force of friction. Indeed, as we showed in class if the applied force is less than a certain value no motion occurs and we talk of static friction (f_s).

Note: friction always opposes motion





Recall the experiment we did in class. We slowly increased F_{app} and since no motion occurred we said $\underrightarrow{f_s} = -F_{app}$. That means that as long as there is no motion of f vs. F_{app} forms a straight line of slope 1. Finally, sliding starts because f_s has a maximum value. That is

$$f_s \le (\mu_s N_R) \qquad\qquad \underrightarrow{f_s} \perp \underrightarrow{N_R}$$

where μ_s is called coefficient of static friction. μ_s is determined by the properties of the two surfaces. If $f_{app} > \mu_s N_R$, sliding begins but frictional force in NOT zero. It is given by

$$f_k = \mu_k N_R$$

μ_k is called coefficient of kinetic friction.

Note: $\underrightarrow{f} \perp \underrightarrow{N_R}$, $\underrightarrow{f}$ always opposed $\underrightarrow{F_{applied}}$