Lengths at Play-Length, Area, Volume, Angle

Since Physics is a science dependent on measurements, apart from some notable exceptions, all physical quantities have units. For instance, if someone asks you, "how tall are you?" and you reply 6 you have not told them anything, but if you say 6 feet, then the enquirer knows precisely what you mean (provided, of course, he/she was raised in a culture where a foot is an accepted measure of length, we return to this in the sequel).

A closely allied quantity is what is called a physical DIMENSION. Again every physical quantity can be expressed as a product of a set of fundamental factors Length (L), Time (T), etc, called Dimensions

The above discussion leads us to formulate the cardinal rule for any equation in physics. If we write:

$$A+B=C+D$$

Then every one of the quantities A, B, C, and D must have the SAME UNITS and of course SAME DIMENSIONS: NEVER WRITE AN EQUATION WHICH IS DIMENSIONALLY INCORRECT. We will emphasize this at every step. No matter how elegant an equation, if it is dimensionally incorrect it will fetch a "ZERO" in an exam.

So, now we start from scratch and begin to build a description of the universe. Since it occupies space the very first quantity we invoke is a measure of the extent of space along a line segment. Right away, we should begin by building a

MASTER TABLE

→ PHYSICAL QUANTITY	DIMENSIONS	UNIT	s/v

(the last column distinguishes scalar/vector and the full significance will develop later)

A line segment spans the extent of space in one space dimension. Let us see the kinds of lengths we will encounter- The SI (systems international) unit is the meter, which is the distance between two scratches on a bar of metal. (Technically, these days it is defined in terms of light wavelengths) However, we want to gauge it from everyday experience. The easiest way to get a feel for it is to note that the typical height of a human being is in the neighborhood of 1.5m to 2m.

Starting with 1m if we go to shorter lengths we get 1 centimeter(cm)=$10^{-2}\,m$ [about the diameter of a finger] 1 micrometer (micron, μm)=$10^{-6}\,m$ [roughly the diameter of human hair] 1 nanometer (nm)= $10^{-9}\,nm$ [about ten times the diameter of the hydrogen atom) 1femtometer (fm)=$10^{-15}\,m$ [diameter of a nucleus].

When we envisage lengths larger than 1m it is useful to keep in mind some useful conversions since in the U.S. we are not customarily using SI units.

Thus One inch=2.54cm
One foot=30.48cm
One Yard=91.44cm
One Mile=1609m=1.609 kilometers

The range of lengths is again very large

Typical city 30km

Country 1000km

Radius of Earth about 6400km

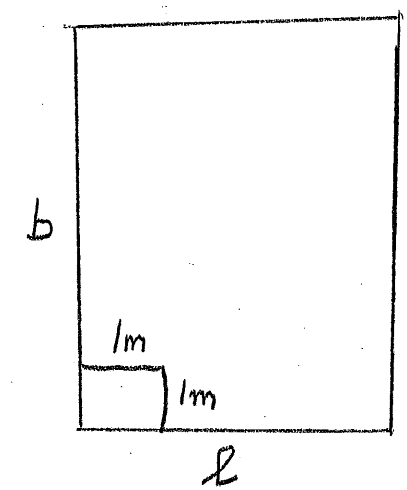
Distance to moon about 400,000km

Distance to Sun $1.5 \times 10^8\,km$

Distance to one of the farthest objects $10^{23}\,m$

So in terms of going from the smallest to the largest, the lengths vary by 10^{38}, a huge span indeed.

Once you have length you can create an area by moving the length parallel to itself you are essentially summing $(l \times b)$ squares each of area $(1 \times 1)\,m^2$.



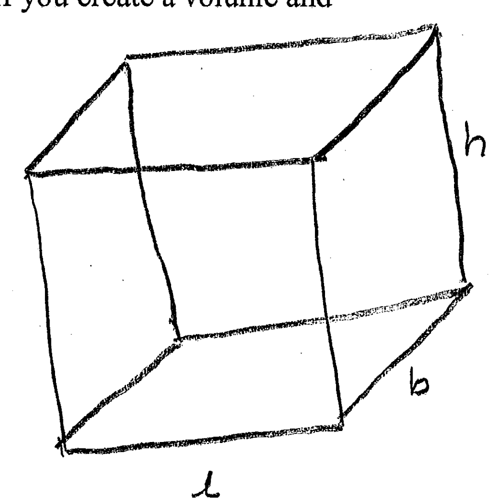
$$A = l \times b$$

Immediately, note the implications of the cardinal rule: You cannot equate an area to a length.

Next, if you move the area parallel to itself you create a volume and

so Volume

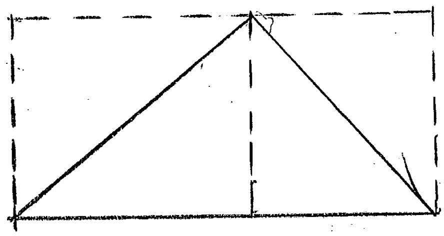
V=lbh
because again you are summing V number of cubes, each of Volume $1m^3$.



Indeed, with the help of 3 lengths we can fill the entire universe.

Examples:

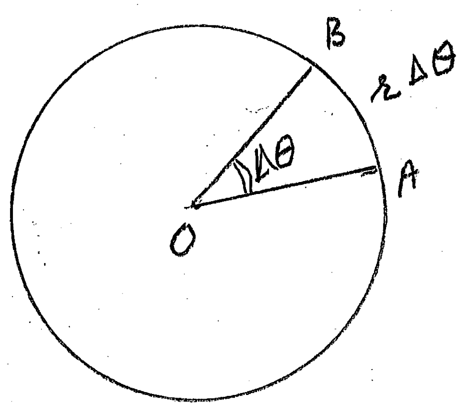
Area ofTriangle



The area of a Δ of base b and height h is essentially one half of the area (see figure) of a rectangle of sides b and h so

$$\Delta Area = \frac{1}{2}bh$$

Circle



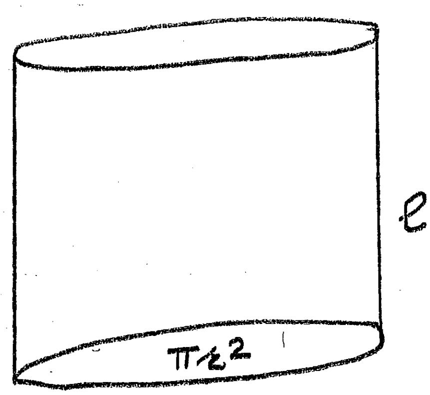
The area of a circle can be calculated by splitting it into a bunch of $\Delta's$ (see figure)

$$\text{Area of } \Delta OAB = \frac{1}{2} \times r\Delta\vartheta \times r = \frac{1}{2} r^2 \Delta\vartheta$$

$$\text{Area of circle} = \frac{1}{2} r^2 \sum \Delta\vartheta$$

$\sum \Delta\vartheta = 2\pi$ So area of circle is πr^2

Volume of Cylinder



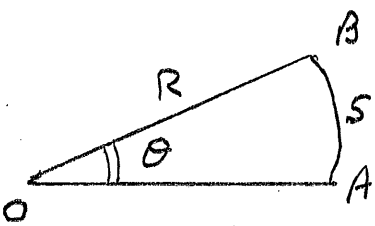
of radius r and length l can be created by moving a circle of area πr^2 parallel to itself so $V_{cyl} = \pi r^2 l$

[Incidentally, if you unfold the surface its area will be ($2\pi rl$)]

Next, still using length as the only dimension we can talk of angle (ϑ) as a measure of the inclination between two lines. Let $OA = r$

Rotate OA by the amount

ϑ about one end (O), the

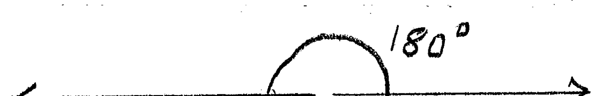


other end A moves from A to B along a circular arc of length S. The angle ϑ which measures the inclination between OA and OB is defined by

$$\vartheta = \frac{S}{R}$$

The unit value requires S=R and is called a radian

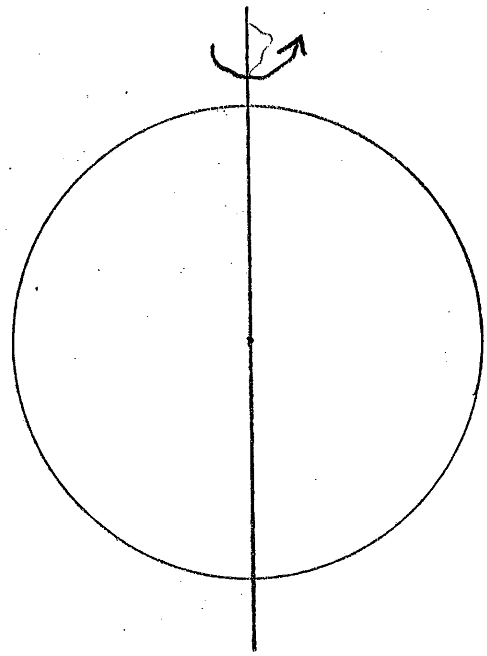
The ratio of the circumference of a circle to its diameter is π radians and if we recall that in common parlance the angle between two antiparallel lines is 180°



we can write $180^\circ = \pi$ radians

Sphere

You can generate a sphere by rotating a circle about one of its diameters (figure)



It turns out that

volume of sphere= $\frac{4\pi}{3} r^3$

surface areas of sphere= $4\pi r^2$

Angle

Question Which is bigger, the sun or the moon?

It is interesting to note that when we look at the "angular width" they are nearly equal.

Diameter of moon $\cong$ 3200km
Distance to Moon=400,000km

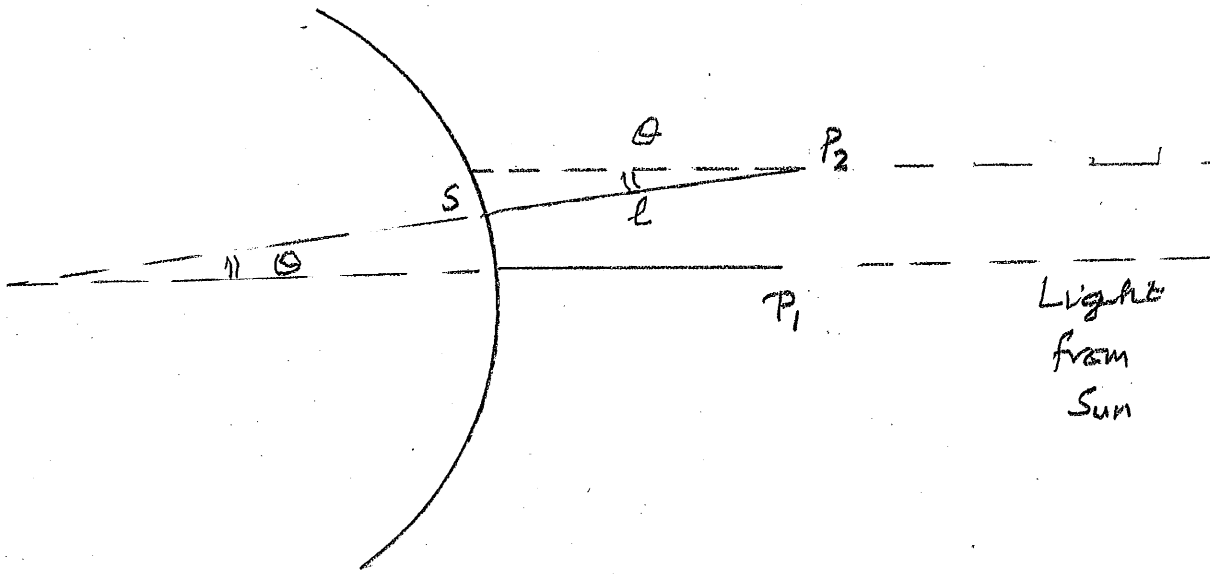
$$\Delta \vartheta_{Moon} = \frac{3200}{400,000} = 8 \times 10^{-3} \text{ radian}$$

Diameter of Sun= $1.4 \times 10^5 \, km$
Distance to Sun= $2 \times 10^8 \, km$

$$\Delta \vartheta_{Sun} \cong 7 \times 10^{-3} \text{ radian}$$

You can do the following experiment:
On a full moon night, take a dime and measure how far it must be from your eye so you can "cover" the moon completely.

Question: Devise a simple experiment to estimate the radius of the Earth schematically, we can draw the picture below when two amateur physicists take on this investigation



Note: R_E is enormous compared to l

When Sun is vertically above P_1 its shadow has no size, but for P_2 the size is S. The angle $\vartheta = \frac{s}{l} - \frac{d}{R_E}$ so knowing d you can estimate R_E.