Kinematics – Description of Motion in One Dimension [Along x-axis]. (Point Particles)

Definitions

Position Vector:

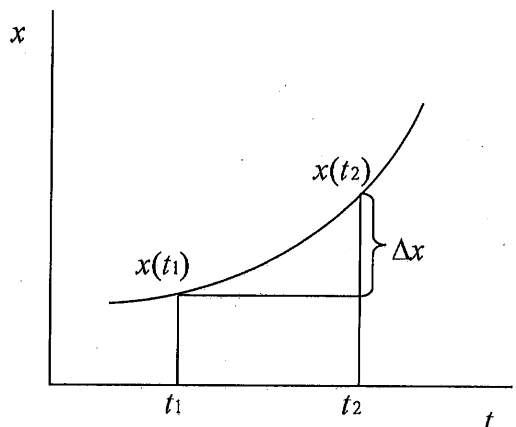
$$\underset{\to}{x}(t) = A\hat{x} \quad or \quad -A\hat{x}$$

A is magnitude
$+\hat{x}$ vector points right
$-\hat{x}$ vector points left

Displacement Vector: Measures change of position

$$\Delta\underset{\to}{x} = \underset{\to}{x}(t_2) - \underset{\to}{x}(t_1)$$

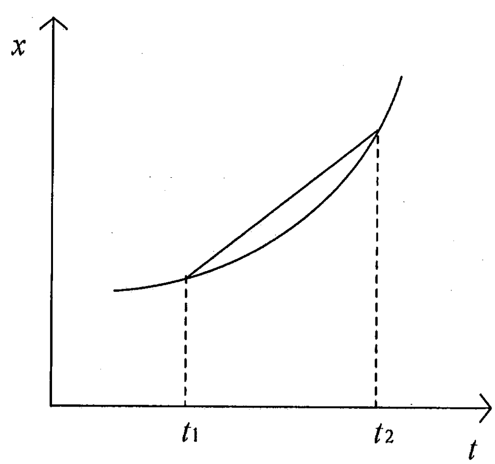
Here, $\Delta\underset{\to}{x}$ is along $+\hat{x}$



Average Velocity Vector: Measures rate of change of position with time over a finite time interval $(t_2 - t_1)$

$$<\underset{\to}{v}> = \frac{x(t_2) - x(t_1)}{(t_2 - t_1)}\hat{x}$$

It is the slope of the chord



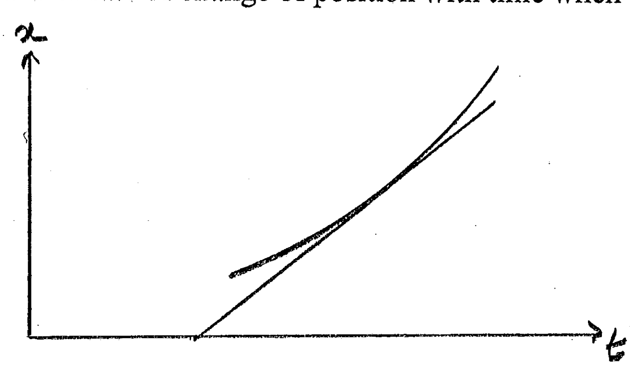
Instantaneous Velocity Vector: Measures rate of change of position with time when time interval goes to zero

$$\Delta t \to 0$$

$$\Delta\underset{\to}{x} \to 0$$

$$\underset{\to}{v} = \lim_{\Delta t \to 0} \frac{\Delta\underset{\to}{x}}{\Delta t}$$

Slope of tangent to x vs. t graph



Average Acceleration Vector: Measures rate of change of velocity vector during a finite time interval.

$$<\underset{\to}{a}>=\frac{\underset{\to}{v}(t_2)-\underset{\to}{v}(t_1)}{(t_2-t_1)}$$

Slope of chord in v vs. t graph

Instantaneous Acceleration Vector: Measures rate of change of velocity vector when time interval goes to zero

$$\Delta t \to 0$$

$$\Delta \underset{\to}{v} \to 0$$

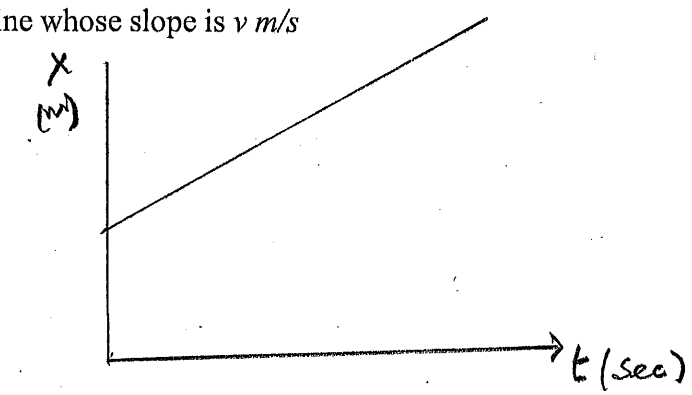
$$\underset{\to}{a} = \lim_{\Delta t \to 0} \frac{\Delta \underset{\to}{v}}{\Delta t}$$

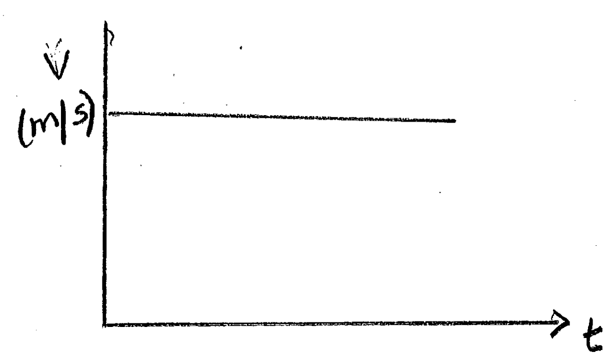
UNIFORM MOTION

$$\underset{\to}{a} = 0$$

$\underset{\to}{v} = v\hat{x}$ is constant

In this case x vs. t graphs will be straight line whose slope is v m/s



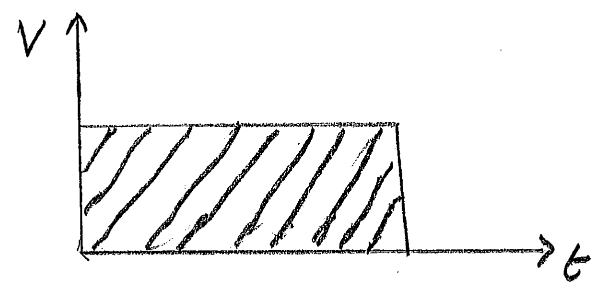


Since v measures change in x every second, a table of Δx vs. t will look like

t (sec)	Δx (m)
0	0
1	v
2	$2v$
3	$3v$
4	$4v$

That is Δx is equal to area under v vs. t graph. In t seconds

$$\Delta x = vt$$



To write down x at t seconds, we must know where object was at $t = 0$ and get uniform motion equation

$$\underset{\to}{x}(t) = \underset{\to}{x}(0) + \underset{\to}{v}t = (x_i + vt)\hat{x} \qquad \longrightarrow (1)$$

$$x_i = \text{initial position}$$

So the rule is: To calculate $\underset{\to}{x}(t)$ add the area under $\underset{\to}{v}$ vs. t graphs to the value of $\underset{\to}{x}$ at $t = 0$.

<u>Next:</u> MOTION WITH CONSTANT ACCELERATION

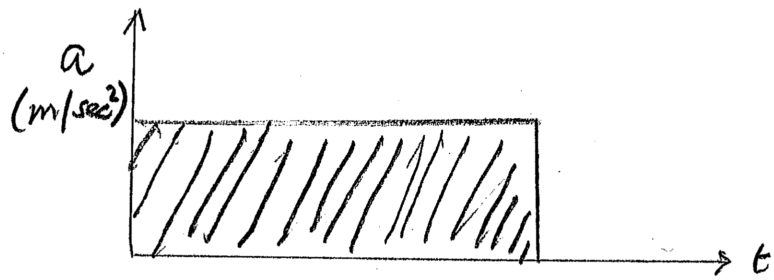
$$\underset{\to}{a} = a\hat{x} \qquad (2)$$

Now $\underset{\to}{v}$ is <u>NOT</u> CONSTANT. Indeed, since a measure change of $\underset{\to}{v}$ every second v vs. t graphs must look like

Now $\underset{\rightarrow}{v}$ is changing by *a m/s* every second so table of Δv vs. *t* must be

t (sec)	Δv (m/s)
0	0
1	a
2	$2a$
3	$3a$
4	$4a$

Change of v during *t* seconds is area under *a* vs. *t* graph.



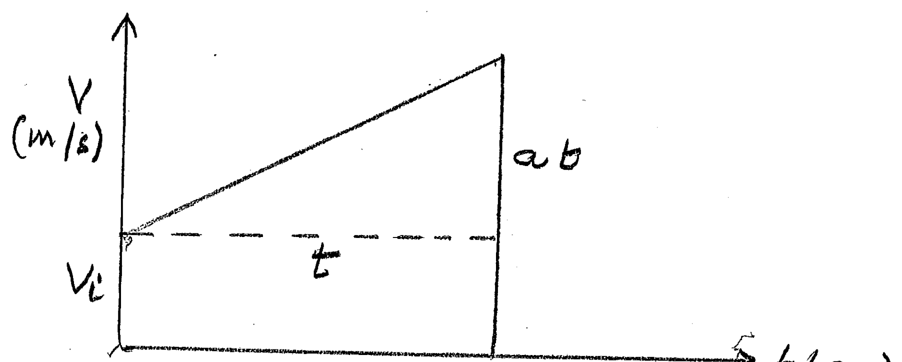
And again to write $\underset{\rightarrow}{v}$ at any time *t* we must know $\underset{\rightarrow}{v}$ at $t = 0$ and write for constant acceleration

$$\underset{\rightarrow}{v}(t) = (v_i + at)\hat{x} \qquad \longrightarrow (3)$$

$v_i\ \hat{x}$ is initial velocity

I To calculate x as a function of t we can proceed in two ways:
Use the rule written under Eq(1). Draw graph of Eq(3) then change of x is area under v vs. t graph

$$\Delta x = v_i t + \frac{1}{2}at^2$$



Hence $$\underset{\rightarrow}{x}(t) = \left(x_i + v_i t + \frac{1}{2}at^2\right)\hat{x} \qquad \longrightarrow (4)$$

II We can use (3) to calculate average velocity between *o* and *t* since v is increasing linearly with time

$$< v >= \frac{v_i + v_i + at}{2} = v_i + \frac{at}{2}$$

Displacement $\Delta x = \left(v_i + \frac{at}{2} \right) t$

and again yield Eq(4)

To Summarize the kinematic equations are

$$\underset{\to}{a} = a\hat{x} \qquad (2)$$

$$\underset{\to}{v}(t) = (v_i + at)\hat{x} \qquad (3)$$

$$\underset{\to}{x}(t) = \left(x_i + v_i t + \frac{1}{2} a t^2 \right) \hat{x} \qquad (4)$$

Eqs(3) and (4) can be combined to yield a useful relation between magnitudes of v and x

From (3) $\quad t = \frac{v - v_i}{2}$

Substitute in (4)

$$x = x_i + v_i \left(\frac{v - v_i}{a} \right) + \frac{1}{2} a \left(\frac{v - v_i}{a} \right)^2$$

$$= x_0 + \frac{v^2 - v_i^2}{2a}$$

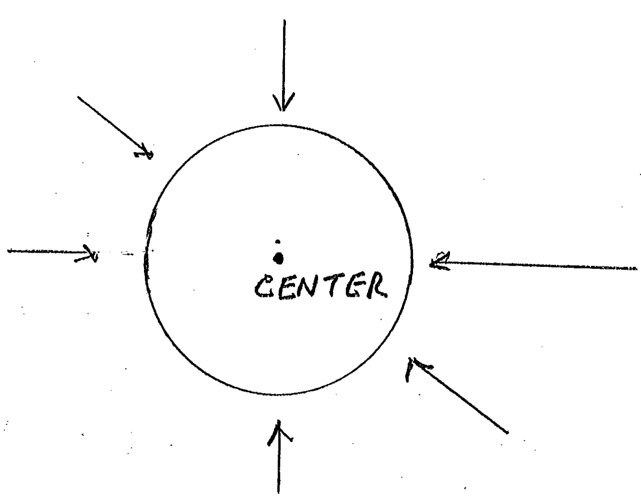
Or

$$v^2 = v_i^2 + 2a(x - x_i) \qquad \longrightarrow (5)$$

Eq(5) is useful when you know position and not time.

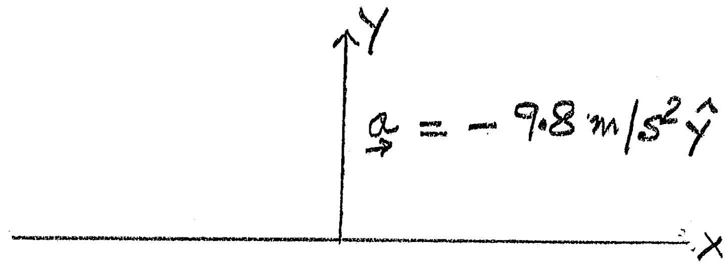
FREE FALL

So, why are we so interested in discussing motion where $\underset{\to}{a}$ is a constant? The reason is that near the surface of the Earth every unsupported object has a constant acceleration of about $9.8\, m/s^2$ directed along the radius of the Earth and pointing toward the center. – acceleration due to gravity



Locally, we pretend that the Earth is flat, choose coordinate system where x is along horizontal y along vertical with positive up and therefore write that the acceleration due to gravity is

$$\underset{\to}{a} = -9.8\, m/s^2\, \hat{y} \tag{-(6)}$$



Now we can use Eqs(3),(4), and (5) for motion along y and write

$$\underset{\to}{v} = (v_i - 9.8t)\hat{y} \qquad \text{-(7)}$$

$$\underset{\to}{y} = (y_i + v_i t - 4.9t^2)\hat{y} \qquad \text{(8)}$$

$$v^2 = v_i^2 - 19.6(y - y_i) \qquad \text{-(9)}$$

Notes

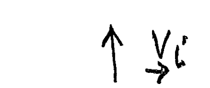
1. It is very important to note that if you throw a ball straight up or straight down the only quantity you can control is its initial velocity v_i. Once it leaves your hand the motion is controlled only by the Earth via Eqns(6) through (9). THE ACCELERATION IS THE SAME AT ALL TIMES DURING THE FLIGHT OF THE BALL.

2. Eqns(6) through (9) apply for free fall on the moon or any other planet. The only difference is that the magnitude of $\underset{\to}{a}$ is not the same as it is on Earth. For instantce, on the moon

$$\underset{\to}{a} = -1.63\, m/s^2\, \hat{y} \qquad \text{(Moon)}$$

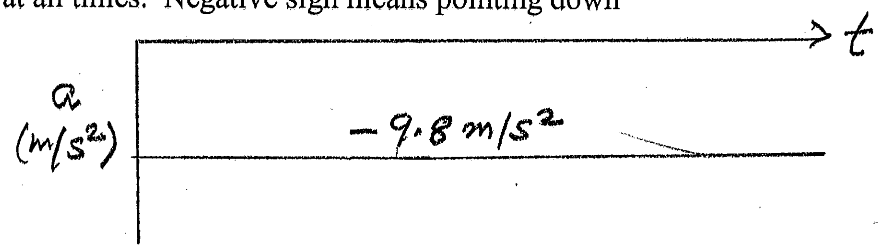
EXAMPLE

Let $\underset{\to}{v} = +v_i\hat{y}$ $\quad y_i = 0$. That is, at $t = 0$ an object is thrown straight up with a velocity of $+v_0\, m/s^2\, \hat{y}$ starting from the ground ($y_i = 0$). It will go up to some height. Turn around and come back to ground according to Eqs(5) through (9). We can plot its acceleration, velocity and position as a function of time.



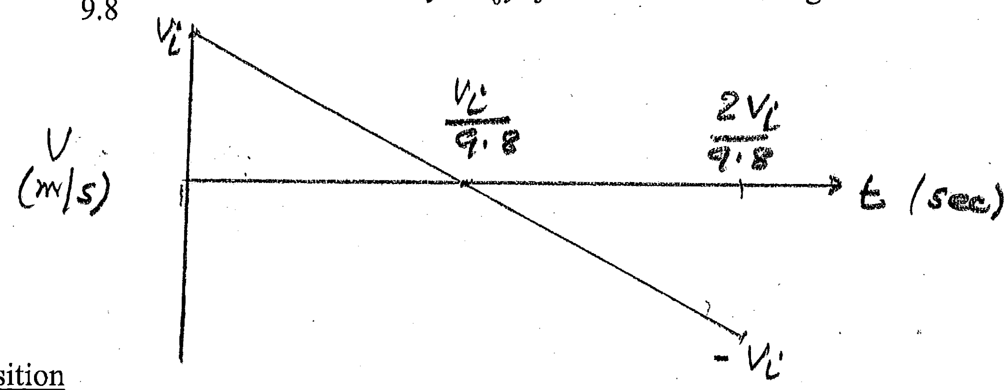
Acceleration

Constant at all times. Negative sign means pointing down



Velocity

Reaches highest point in $\frac{v_i}{9.8}$ seconds. At that point, velocity is zero so it stops rising. Returns to Earth in $\frac{2v_i}{9.8}$ seconds and has velocity $-v_0\hat{y}$ just before it hits the ground.



Position

At highest point, $v = 0$ so from Eq(9) $y_{top} = \frac{v_i^2}{19.6}$.

Combines Flight Picture