CONSERVATION PRINCIPLES

CPM: CONSERVATION OF MASS
IN A CLOSED (NO EXCHANGE OF MATTER WITH SURROUNDINGS) SYSTEM THE TOTAL MASS IS CONSTANT.

CPE: CONSERVATION OF ENERGY
IN AN ISOLATED SYSTEM TOTAL ENERGY IS CONSTANT. IN OUR PRESENT DISCUSSION WE TALK OF MECHANICAL ENERGY.

CPP: CONSERVATION OF LINEAR MOMENTUM
IF THERE IS NO EXTERNAL FORCE PRESENT, THE TOTAL (VECTOR) LINEAR MOMENTUM OF A SYSTEM IS CONSTANT.

CPL: CONSERVATION OF ANGULAR MOMENTUM
IF THERE IS NO EXTERNAL TORQUE THE TOTAL (VECTOR) ANGULAR MOMENTUM IS CONSTANT.

CPE

The ingredients required to state the conservation principle for mechanical energy are:

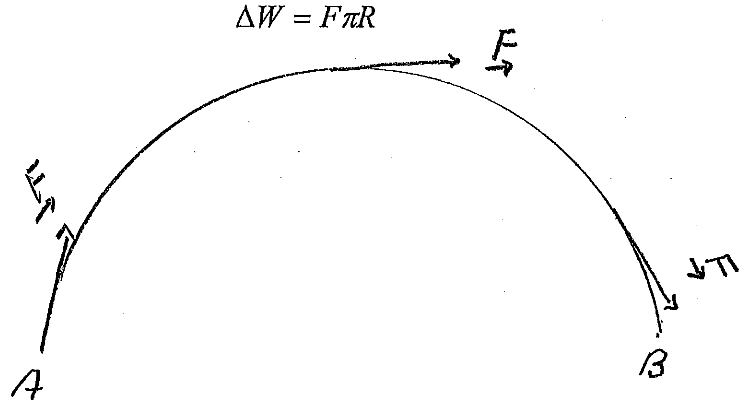
MECHANICAL WORK

If the point of application of a constant force $\underset{\rightarrow}{F}$ is displaced through an amount $\underset{\rightarrow}{\Delta S}$ the amount of work done is

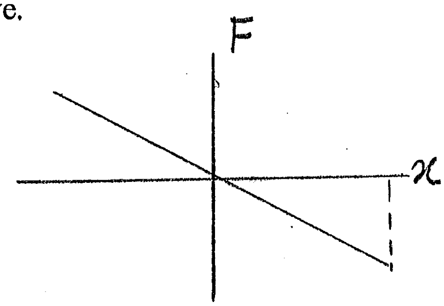
$$\begin{aligned}\Delta W &= \underset{\rightarrow}{F} \bullet \underset{\rightarrow}{\Delta S} \\ &= F\Delta S \cos(\underset{\rightarrow}{F}, \underset{\rightarrow}{\Delta S}) \\ &= F_x \Delta_x + Fy\Delta y + F_z \Delta z\end{aligned}$$

so ΔW is the "DOT" product of the force vector and the displacement vector. Notice, that we are multiplying the component of $\underset{\rightarrow}{F}$ along $\underset{\rightarrow}{\Delta S}$ by ΔS to get the work done. No work is DONE if $\underset{\rightarrow}{F} \perp \underset{\rightarrow}{\Delta S}$. Also, note that $\underset{\rightarrow}{\Delta S}$ measure the total displacement of $\underset{\rightarrow}{F}$. For example, AB is half a circle of radius R. If you apply a force $\underset{\rightarrow}{F}$ which is tangent to ⊕ at every point, total work done is

$$\Delta W = F\pi R$$



If F_x, varies with x, work done is are under F_x vs. x curve.



Ex: Spring Force $\underrightarrow{F} = -kx\hat{x}$

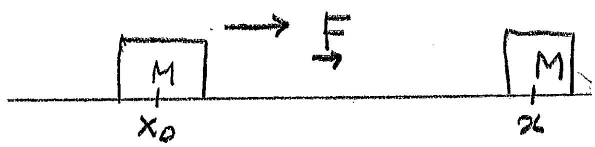
Work done by spring in going from x to zero is

$$\Delta W = \frac{1}{2}kx^2$$

KINETIC ENERGY

MECHANICAL WORK STORED IN GIVING A FINITE SPEED TO A MASS-WORK STORED IN MOTION

Object at rest as x_0. Apply force $\underrightarrow{F} = F\hat{x}$, keep force on until object reaches x.



$$\Delta W = F(x - x_0)$$
$$= Ma(x - x_0)$$

But we know that $v^2 = v_0^{\ 2} + 2a(x - x_0)$

$$v_0 = 0, \quad v = 2a(x - x_0)$$

So $\Delta W = \frac{1}{2}Mv^2$

After $\underrightarrow{F}$ turned off, M moved on with speed v. We have stored kinetic energy

$$K = \frac{1}{2}Mv^2$$

in its motion.

POTENTIAL ENERGY (See attached note)

MECHANICAL WORK STORED IN A SYSTEM WHEN IT IS ASSEMBLED IN THE PRESENCE OF A PREVAILING CONSERVATIVE FORCE $\underrightarrow{F}$.

Change in potential energy

$$\Delta P = -\underrightarrow{F} \bullet \underrightarrow{\Delta S}$$

(NEVER FORGET THE "MINUS" SIGN! WHY?)

We have two conservative forces available

i) $\underrightarrow{W_g} = -Mg\hat{y}$, so taking $P_g = 0$ at Earth's surface we write

$$P_g(y) = Mgy$$

as the potential energy of the Mass-Earth system.

ii) $\underrightarrow{F_{sp}} = -kx\hat{x}$ so

$$P_{sp}(x) = \frac{1}{2}kx^2$$

Now we have all three W, K, and P we can write <u>CPE</u>.

<u>ISOLATED SYSTEM</u>

$$K_f + P_f = K_i + P_i$$

$i =$ initial state
$f =$ final state

<u>EXTERNAL WORK INCLUDED</u>

$$K_f + P_f = K_i + P_i + W_{NCF}$$

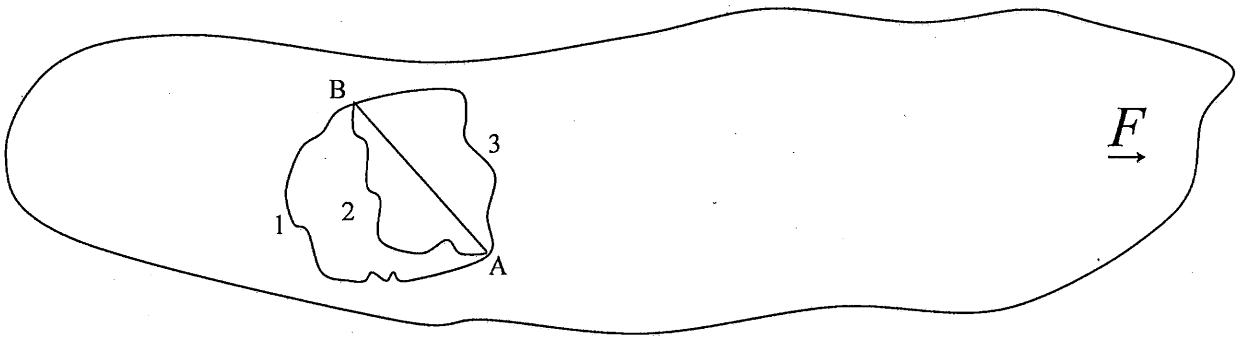
NCF refers to Non-Conservative force. (friction, force applied by your hand etc.)

<u>Note:</u> if NCF is f_k, W_{NCF} IS ALWAYS NEGATIVE!!!

Potential Energy (P) presents a greater conceptual challenge.

P is the mechanical work stored in a system when it is prepared (or put together) in the presence of a prevailing conservative force.

Suppose we have a region of space in which there is a prevailing force (weight near Earth's surface comes to mind). That is, at every point in this region an object will experience a force. Let the object be at point B [First, notice that you can't let the object go as $\underrightarrow{F}$ will immediately cause $\underrightarrow{a}$ and object will move].



To define P at B we have to calculate how much work was needed to put the object at B in the presence of $\underrightarrow{F}$. Le us pick some point A, where we can claim that P is known, and calculate the work needed to go from A to B. As soon as we try to do that we realize that the only way we can get a meaningful answer is if the work required to go from A to B is independent of the path taken. So our prevailing force has to be special. Such a force is called a CONSERVATIVE FORCE – WORK DONE DEPENDS ONLY ON END-POINTS AND NOT ON THE PATH TAKEN.

If that is true we have a unique answer

$$\Delta W_1 = \Delta W_2 = \Delta W_3 = \Delta W_{AB}$$

and we can use this fact to calculate the change in P in going from A to B

$$\Delta P_{AB} = -\underrightarrow{F} \bullet \underrightarrow{\Delta S_{AB}}$$

NOTE THE –SIGN: It comes about because as stated above we cannot let the object go. In fact, the displacement from A to B must be carried out in such a way that the object cannot change its speed (if any). That is, we need to apply a force $-\underrightarrow{F}$ to balance the ambient $\underrightarrow{F}$ at every point. The net force will become close to zero at all points. ΔP_{AB} is work done by $-\underrightarrow{F}$.

So when $\underrightarrow{F}$ is conservative ΔP_{AB} is unique. In the final step we can choose A such that $P_A = 0$. Then $P_B = -\underrightarrow{F} \bullet \underrightarrow{\Delta S_{AB}}$.