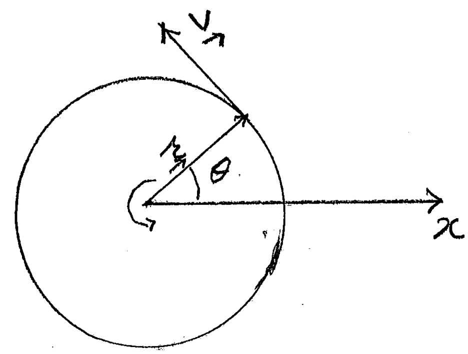
CIRCULAR MOTION WHEN SPEED IS NOT CONSTANT

Up to now we have been considering circular motion where the speed was constant so we could define period T, and write $S = \frac{2\pi R}{T}$



$\vec{r} = R\hat{r}$	rotating by ω radians per second
$\vec{v} = R\omega\hat{\tau}$	rotating by ω radians per second
$\vec{a}_c = -R\omega^2\hat{r}$	rotating by ω radians per second
$\vec{\omega} = \pm\frac{\Delta\Theta}{\Delta t}\hat{n}$ $\longrightarrow$	constant

If we want to think of how the angle Θ changes with time we can construct a table, let $\omega = 0.1\ rad/s$ and write $\vec{\Theta} = (\Theta_i + \omega t)\hat{n}$ where Θ_i is angle at $t = 0$ exactly as we wrote $\vec{x} = (x_i + vt)\hat{x}$ sometime ago.

time (sec)	$\Delta\Theta$ (rad)
1	0
2	0.1
3	0.2
t	$0.1t$

Next, we want to consider a situation where speed is not constant. This means that the angular speed is also not constant. We will not change the direction of $\vec{\omega}$, only its magnitude and define angular acceleration vector

$$\vec{\alpha} = \frac{\Delta\vec{\omega}}{\Delta t} \qquad \left[L^{\circ}T^{-2} \quad rad/s^2\ vector\right]$$

and α measures the change in ω per second so now

$$\vec{\omega} = \pm(\omega_i + \alpha t)\hat{n} \qquad \left|Compare \quad \vec{v} = (v_i + at)\hat{x}\right|$$

where w_i is angular velocity at $t = 0$. And following the same steps as before

$$\vec{\Theta} = \pm\left(\Theta_i + \omega_i t + \frac{1}{2}\alpha t^2\right)\hat{n} \qquad \left[Compare \quad \vec{x} = \pm\left(x_i + v_i t + \frac{1}{2}\alpha t^2\right)\hat{x}\right]$$

So kinematic equations are

Linear Motion (one dimension)	Angular Motion (rotations about $\hat{n}$)
x	Θ
$\vec{a} = a\hat{x}$	$\vec{\alpha} = \alpha\hat{n}$
$\vec{v} = (v_i + at)\hat{x}$	$\vec{\omega} = \pm(\omega_i + \alpha t)\hat{n}$
$\vec{x} = \pm\left(x_i + v_i t + \frac{1}{2}\alpha t^2\right)\hat{x}$	$\vec{\Theta} = \pm\left(\Theta_i + \omega_i t + \frac{1}{2}\alpha t^2\right)\hat{n}$
$v^2 = v_i^2 = 2a(x - x_i)$	$\omega^2 = \omega_i^2 = 2\alpha(\Theta - \Theta_i)$

To cause an acceleration $\vec{a}$, Newton taught us that we must provide a force at that point at that time

$$M\vec{a} = \Sigma \vec{F_i} \qquad \text{(at that point at that time)}$$

What do we need to cause angular acceleration $\vec{\alpha}$? A new physical agency which we will develop next.

Before we go there let us note that we still have

$$\vec{r} = R\hat{r}$$
$$\vec{v} = R\omega\hat{\tau}$$
$$\vec{a_c} = -R\omega^2\hat{r}$$

but they no longer rotate at constant rates and the magnitudes of v and $\vec{a_c}$ are now varying with time. Indeed now in addition to centripetal acceleration we have TANGENTIAL ACCELERATION

$$\vec{a_t} = R\frac{\Delta\omega}{\Delta t}\hat{\tau} = R\,\alpha\,\hat{\tau}$$

and in accord with Newton's Law we not only need a centripetal force

$$\vec{F_C} = -MR\omega^2\hat{r}$$

but also a tangential force

$$\vec{F_t} = Ma_t\hat{\tau}$$

which leads to a new physical agency.