Week 6-Problems

6-1 Suppose you are driving a car at 20m/s on a straight road and you have a pendulum hanging inside. The pendulum will hang vertically. Now, if the road curves on a radius of 100 meters and your speed is the same, will the pendulum still hang vertically? Justify your answer.

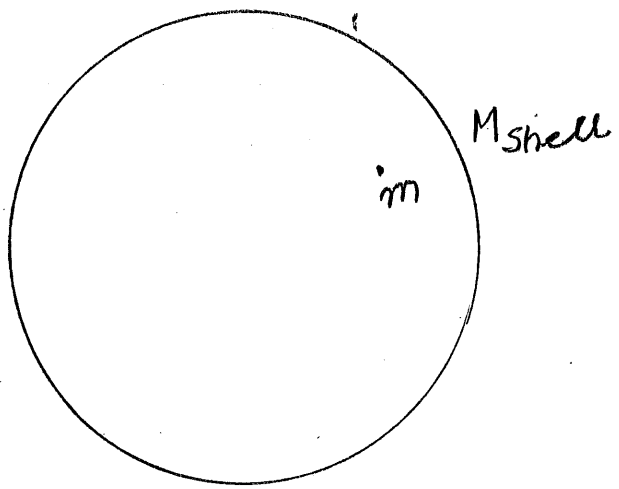
6-2 If the answer to problem 1 is No, "it is not vertical any more," what angle doe sit make with the vertical?

6-3 On Earth, why does a pendulum hang vertically only at the poles and the equator?

6-4 We have shown that because of the rotation about its axis, only systems located at the poles of Earth can be treated as inertial systems. Can you think of a reason why even this conclusion is not strictly correct?

6-5 Newton's universal Gravitational force between two point's masses M_1 and M_2 is written as $\underrightarrow{F_G} = -\frac{GM_1M_2\hat{r}}{r^2}$. Why is there a minus sign on the right side of the equation?

6-6 A point mass m is located inside a spherical shell of mass M_{shell}. What is the force experienced by the shell? Why?



6-7 Newton showed that inside a hollow sphere (mass M), the gravitational force is zero. While outside it is $\underrightarrow{F_G} = -\frac{GMm}{r^2}\hat{r}$ for a point, mass m located at a distance r from the center of M. Using these show that for a uniform solid sphere of radius R and Mass M, the gravitational force on a point mass located a distance r from the center of the sphere is: $\underrightarrow{F_G} = -\frac{4\pi}{3} G \rho m r \hat{r}$ r<R

$$\underrightarrow{F_G} = -\frac{GMm}{r^2}\hat{r} \qquad r>R$$

Where $\rho = \frac{M}{\frac{4\pi}{3}R^3}$

6-8 A geosynchronous (or geostationary) satellite is located such that it is vertically above a given point on Earth (that is, if you stand at that point and look up you will see the satellite at all times). (i) Where would you locate such a satellite, (ii) what is the period?

6-9 The period and radius of the orbit of our moon are about 27 days and 4×10^5km. The corresponding values for a moon of Jupiter are 3.5days and 7×10^5km. Compute the ratio of the masses of Earth and Jupiter (M_E/M_s).

6-10 The Earth has a mass which is about 81 times the mass of the moon. Where would you locate an object on the Earth-moon line so it experiences no force?

6-11 The sun is moving in a circular orbit of radius 3×10^{17}km about the center of our Galaxy. Assuming that the entire mass of the Galaxy (4×10^{43}kg) is at its center. Calculate the period of the sun and its orbital speed.

6-12 Asuming circular Keplerian orbits, the square of the period (Tp) is proportional to the cube of the radius (Rp). That is

$$Tp^2 \alpha Rp^3$$

If you increase Rp would you expect the orbital speed to increase, reduce or remain the same? Why? (The answer to this question has profound consequences leading to the discovery of Dark Matter.)

6-13 If you imagine the Earth to be a sphere of uniform density, at what distances from its center would your weight be $\frac{1}{4}$, $\frac{1}{2}$ of your weight at the surface (R_E = 6400 km). Why?

6-14 The moon, which is satellite of Earth, has a circular orbit radius of about 400,000km and a period of roughly 27 days. Where would you place an Earth satellite so it has a period of 1 (one) day? Why?