Answers– Week 1

1-1. Circumference $= 2\pi r$
$$C_1 = 2\pi r_1$$
$$C_2 = 2\pi r_2$$
So $(C_1 - C_2) = 2\pi (r_1 - r_2)$

1-3. Volume $V = 0.1 cm^3 = 0.1 \times (10^{-2})^3 m^3 = 10^{-7} m^3$
$V = \pi r^2 t$

$$t = \frac{10^{-7}}{\pi \times (2.5)^2} = 0.05 \times 10^{-7} m = 5 \times 10^{-9} m$$

1-5. $1\ mi = 1.609 km$
$R_E = 4000 mi$
$\quad = 6436 km$
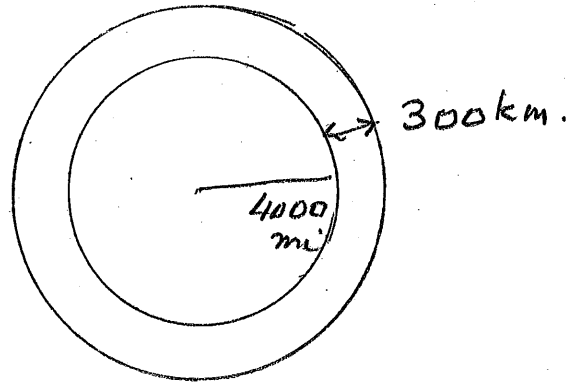Radius of orbit
$\quad r = 6736 km$
Circumference
$\quad C = 2\pi r = 4.23 \times 10^4 km = 4.23 \times 10^7 m$

300 km.

4000 mi.

1.7. $r = (10 \pm 0.2) cm = (r \pm \delta r) cm = r\left[1 \pm \dfrac{\delta r}{r}\right]$

$$\frac{\delta r}{r} = \frac{0.2}{10} \quad (2\%)$$

$$d = 2(r \pm \delta r) = 2r\left[1 \pm \frac{\delta r}{r}\right]$$

uncertain in d also $\dfrac{\delta r}{r} = \dfrac{0.2}{10}$ (2%)

Area $= \pi (r \pm \delta r)^2 = \pi\left[r^2 \pm 2r\,\delta r + (\delta r)^2\right]$

Area $= A = \pi r^2\left[1 \pm \dfrac{2\delta r}{r} + \left(\dfrac{\delta r}{r}\right)^2\right]$ $\qquad \left(\dfrac{\delta r}{r}\right)^2 \ll 1$

Uncertainty in A $\quad \dfrac{2\delta r}{r} = \dfrac{0.4}{10} = 4\%$

Volume $V = \dfrac{4\pi}{3}(r \pm \delta r)^3$

$$= \frac{4\pi}{3}\left[r^3 \pm 3r^2\delta r + 3r(\delta r)^2 \pm (\delta r)^3\right]$$

$$= \frac{4\pi}{3}r^3\left[1\pm\frac{3\delta r}{r}\pm\ldots\right] \qquad \left(\frac{\delta r}{r}\right)^2,\left(\frac{\delta r}{r}\right)^3 \ll 1$$

Uncertainty in vol. $\dfrac{3\delta r}{r} = \dfrac{0.6}{10} = 6\%$

Notice: how the percentage error increases with the exponent.

1-11. $h^2 = o^2 + b^2$

$\operatorname{Sin}\theta = \dfrac{O}{h}$

$\operatorname{Cos}\theta = \dfrac{b}{h}$, $\operatorname{Sec}\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\dfrac{o^2+b^2}{h} = \dfrac{h^2}{h^2}$ $\qquad\qquad \dfrac{o^2}{h^2}+\dfrac{b^2}{h^2} = 1$

$\operatorname{Sin}^2\theta + \operatorname{Cos}^2\theta = 1$

$\dfrac{\operatorname{Sin}^2\theta + \operatorname{Cos}^2\theta}{\operatorname{Cos}^2\theta} = \dfrac{1}{\operatorname{Cos}^2\theta}$

$\tan^2\theta + 1 = \operatorname{Sec}^2\theta$
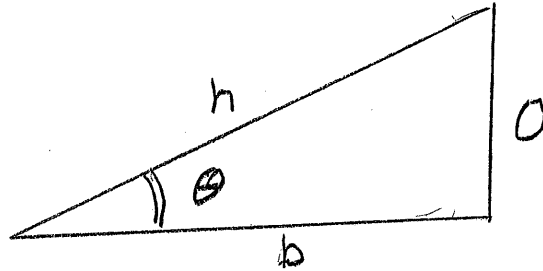
1-13. a) $\vec{V} = +5\text{m }\hat{x}$

b) $\vec{V} = -3m\,\hat{y}$

c) $\vec{V} = +3\text{m }\hat{x} - 6\text{m }\hat{z}$

d) $\vec{V} = +1\text{m }\hat{x} - 2m\,\hat{y} + 3m\,\hat{z}$