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Exam I,
Physics 117-Spring 2003, Mon. 3/10/2003
Instructor: Dr. S. Liberati

General Instructions

- There are a total of five problems in this exam.
- All problems carry equal weights.
- Do all the five problems by writing on the exam book (continue to work on the back of each page if you run out of room).
- Write your name (in capital letters) on every page of the exam.
- Purely numerical answers will not be accepted. Explain with symbols or words your line of reasoning. Corrected formulae count more than corrected numbers.

What you can do

- You may look at your text book in taking the exam
- Use a calculator

What you can't do

- Speak with nearby colleagues
- Use any wireless device during the exam

Hints to do well

- Read carefully the problem before to compute. Before to start you must have clear in your mind what you need to arrive to the answer.
- Do problems with symbols first (introduce them if you have to).
  Only put in numbers at the end.
- Check your answers for dimensional correctness.
- If you are not absolutely sure about a problem, please write down what you understand so that partial credit can be given.

Honor Pledge: Please sign at the end of the statement below confirming that you will abide by the University of Maryland Honor Pledge

"I pledge on my honor that I have not given or received any unauthorized assistance on this assignment/examination."

Signature:_______________________________

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Exercise 1

A large boulder rolls off the edge of a cliff with a horizontal speed of 10 m/s. If the cliff is 45 m high

Q-1.1: *How long takes the boulder to hit the ground?*
(Assume $g=10\ m/s^2$ and ignore air resistance)

If we call h the hight of the cliff then the time the boulder takes to fall down is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{90\ \ m}{10\ \ m/s^2}} = 3\ \ s$$

Q-1.2: *How far does it land from the base of the cliff?*

Let's call d the distance that the boulders travels in the horizontal direction before to touch the ground. Then

$$d = v_{horiz} t = 10\ \ \frac{m}{s} \cdot 3\ \ s = 30\ \ m$$

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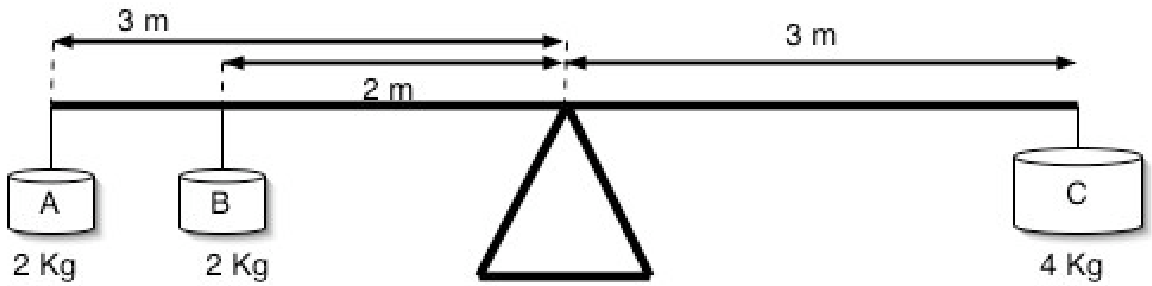
Exercise 2

Look at the following picture



Q-2.1: *Is the system shown above balanced? If not which end will fall? Explain your reasoning.*

The system is balanced if the torques on each side are equal.

But this is not the case, in fact we have:

Right: $\tau_{right} = F_C \cdot R_C = Weight_C \cdot R_C = (4\ Kg \cdot g) \cdot 3\ m = 120\ Nm$

Left: $\tau_{left} = \tau_A + \tau_B = Weight_A \cdot R_A + Weight_B \cdot R_B = (2\ Kg \cdot g) \cdot 3\ m + (2\ Kg \cdot g) \cdot 2\ m = 100\ Nm$

The system is unbalanced.

The side with the larger torque will go down, hence it will be the right side.

Q-2.2: *How would you balance the system by moving just the heaviest weight C? Explain your reasoning.*

The system is balanced if the torques on each side are equal.

So to have equilibrium changing only the position of the weigth C we need that

$\tau_{right} = \tau_{left} = 100\ Nm$

Hence $F_C \cdot R_C = 100\ Nm \Rightarrow R_C = \dfrac{100\ Nm}{(4\ Kg \cdot 10\ m/s^2)} = 2.5\ m$

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Exercise 3

The mass of a certain neutron star is $M_{NS} \approx 4\ 10^5\ M_{Earth}$ and its radius is $R_{NS} \approx 12.5\ 10^{-4}\ R_{Earth}$. (Assume $g_{Earth}=10\ m/s^2$ and Don't be scared by big numbers, use scientific notation)

Q-3.1: *What is the acceleration of gravity on the surface of this condensed, burned-out star?*

Let's label the neutron star quantities NS, then the acceleration of gravity on the surface of the start is $g_{NS} = \dfrac{GM_{NS}}{R_{NS}^2}$

Now we can compute this quantity or by brute force, just using the numbers and a calculator, or by using our knowledge of g_{Earth}

$$g_{NS} = \frac{GM_{NS}}{R_{NS}^2} = \frac{GM_{Earth}\,4\cdot 10^5}{R_{Earth}^2\cdot\left(12.5\cdot 10^{-4}\right)^2} = g_{Earth} \approx 256\cdot 10^9 \approx 256\cdot 10^{10} \quad m/s^2 \quad !!!$$

Q-3.2: *Write the formula for the gravitational acceleration a_{grav} at a distance R from the center of the star and calculate a_{grav} for $R=16\cdot 10^5\cdot R_{NS}$.*

The gravitational acceleration on any body at a distance R from the neutron star is

$$a_{grav} = \frac{GM_{NS}}{R^2}$$

for $R = 16\cdot 10^5\cdot R_{NS}$ we get

$$a_{grav} = \frac{GM_{NS}}{R^2} = \frac{GM_{NS}}{\left(16\cdot 10^5\right)^2 R_{NS}^2} = g_{NS}\frac{1}{256\cdot 10^{10}} = 1\ \frac{m}{s^2}$$

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Exercise 4

A boxcar traveling at 30 m/s approaches a string of 4 boxcars, identical to the first one, sitting stationary on the track. The boxcar collides and links with the stationary cars and the five of them move off together along the track. At any time there are no external forces acting on the boxcars.

Q-4.1: *What is the final speed of the five cars immediately after the collision?*

> We need to use the conservation of momentum (no external forces).
>
> Let's call m the mass of a boxcar.
>
> $$\left.\begin{aligned} p_{initial} &= m \cdot \mathrm{v}_{initial} \\ p_{final} &= 5m \cdot \mathrm{v}_{final} \end{aligned}\right\} \Rightarrow \Delta p = 0 \text{ implies } \mathrm{v}_{final} = \frac{1}{5}\mathrm{v}_{initial} = 6 \ m/s$$

Q-4.2: *What is the change in the total kinetic energy before and after the collision knowing that each cart has a 100 Kg mass. Is the collision elastic?*

> The kinetic energy is in general $KE = \frac{1}{2}M\mathrm{v}^2$
>
> So $\Delta KE = \frac{1}{2}5m\mathrm{v}_{final}^2 - \frac{1}{2}m\mathrm{v}_{initial}^2 = \frac{1}{2}m\left(5 \cdot 36 \ m^2/s^2 - 900 \ m^2/s^2\right) =$
>
> $= 50Kg \cdot \left(-720 \ m^2/s^2\right) \Rightarrow \Delta KE = -36000 \ J$
>
> Clearly some kinetic energy is loss so the collision was not elastic.

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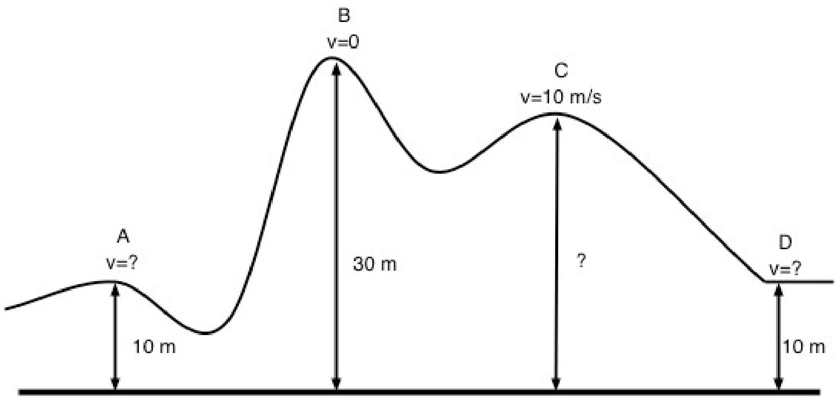
Exercise 5

A cart is riding a rollercoaster as described in the picture



Ignore the friction on the track.
Knowing that at the point B the cart arrives with basically zero speed, calculate

Q-5.1: *The speed of the cart at point A given that the altitude at A is* $h_A = 10\ m$

The basic concept to be used in order to solve this kind of problem is
the conservation of the mechanical energy

$$ME = GPE + KE = mgh + \frac{1}{2}m\mathrm{v}^2$$

The conservation of *ME* is correct given the lack (or negligibility) of friction (see text).

The mechanical energy at A should be equal to the mechanical energy at B.
At B we know that $\mathrm{v}_B = 0$ so there is just gravitational potential energy. So

$$mgh_A + \frac{1}{2}m\mathrm{v}_A^2 = mgh_B \quad \Rightarrow \quad \frac{1}{2}m\mathrm{v}_A^2 = mgh_B - mgh_A = mg(h_B - h_A)$$

$$\Rightarrow \quad \mathrm{v}_A^2 = 2g(h_B - h_A) \quad \Rightarrow \quad \mathrm{v}_A = \sqrt{2g(h_B - h_A)} = \sqrt{2 \cdot 10\ \frac{m}{s^2} \cdot 20\ m} = 20\ \frac{m}{s}$$

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Q-5.2*: The altitude of point C if the speed of the cart there is v_C=10 m/s*

Using that $v_B^2 = 0$ (cart at rest) and the conservation of the mechanical energy, we can write that at point C

$$mgh_C + \frac{1}{2}mv_C^2 = mgh_B \Rightarrow gh_C = -\frac{1}{2}v_C^2 + gh_B \Rightarrow h_C = h_B - \frac{1}{2}\frac{v_C^2}{g}$$

$$h_C = 30m - \frac{1}{2}\frac{100\ \ m^2/s^2}{10\ \ m/s^2} = 30m - 5m = 25m$$

Q-5.3*: The speed of the cart at point D given that the altitude at D is h_D= h_A=10m*

Given that D has the same altitude of A the kinetic energy must be the same so $v_B = v_A = 20\ \ m/s$

Q-5.4: *If the cart has mass 1000 Kg, what is the work that the brakes should do on the cart's wheels in order to stop it just after point D? Note the track is even (no slope) after point D.*

The work needed is equal to the change in kinetic energy

$$W = \Delta KE = \frac{1}{2}mv_D^2 - 0$$

If the cart has mass 1000 Kg then

$$W = \Delta KE = \frac{1}{2}mv_D^2 - 0 = 200000\ \ J = 2\cdot 10^5\ \ J$$