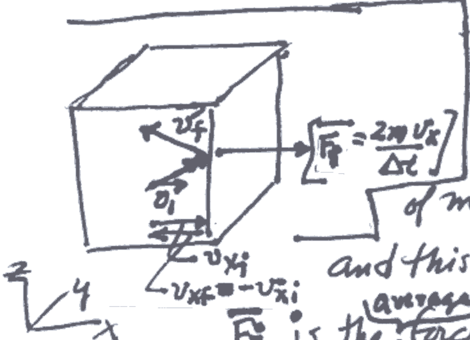
J. GRIFFIN 11/15/04
Physics 117

The Ideal Gas Law: $PV = nRT_A = Nk_BT_A$
and the inference that $T_A \propto$ (KE) of average gas molecule, can be obtained by elementary considerations.
Take the container to be a cube L on each side.
Assume that each gas particle has speed v and all velocity directions are equally probable.

(1) Compute the (AVERAGE) force exerted by the particles hitting the Right side of the cube & Divide this Avg. Force by L^2 = Area of Right Face to obtain: $P = |\vec{F}_{AV}|/L^2$

(2) To compute $|\vec{F}_{AVG}|$, consider one molecule of gas, which has x-component of $\vec{v}$ of $+v_x$ as it hits the RIGHT face. It rebounds elastically with a final x-component of velocity, $-v_x$. Its x-component of momentum is changed by $\Delta p_x = -mv_x - (+mv_x) = -2mv_x$.



and this requires an impulse, $\bar{F}_i \Delta t = -2mv_x$, where $\bar{F}_i$ is the (average) force exerted by the wall on the ith molecule during the collision. (And the force on the face is $\bar{F}_i = +2mv_x$ by Newton IIIrd Law.)

(3) Compute the rate, R, at which molecules strike the right face, assuming that there are N molecules in the box. During a small interval Δt, all of the molecules within $v_x\Delta t$ of the right face which are travelling to the RIGHT (as ½ are at any moment) will hit the face. Therefore $R\Delta t = \frac{N}{2}\cdot\frac{v_x\Delta t}{L}$ molecules hit the RIGHT face during Δt. [The fraction of such molecules is $\frac{v_x\Delta t}{2L}$.]

(4) The average force on the right face during a small interval Δt is the product of $\bar{F}_i$ for one molecule times the No of molecules hitting during Δt: $F_{AVG} = (\bar{F}_i)(\text{No hits}) = \left(\frac{2mv_x}{\Delta t}\right)\left(\frac{N}{2}\frac{v_x\Delta t}{L}\right) = \frac{N}{L}mv_x^2$
& The Pressure is $\frac{F_{AVG}}{L^2} = P = \frac{1}{L^2}\frac{N}{L}mv_x^2 = \frac{1}{V}\cdot N(mv_x^2)$: $PV = N(m\overline{v_x^2}) = kT_A$

(5) In this way $kT_A = \overline{mv_x^2} = \frac{2}{3}\left[\frac{m}{2}(v_x^2 + v_y^2 + v_z^2)\right] = \frac{2}{3}\overline{(KE)}$; Thus, average $\overline{KE}$ of a molecule $\overline{KE} = \frac{3}{2}kT_A$, the GAS LAW follows.