**UNIFORM CIRCULAR MOTION**

\[ |\vec{v}| = \text{speed} = \text{constant} = |\vec{v}_f| = |\vec{v}_i| \]

\[ R = \text{Radius of Circle; } m = \text{mass of object.} \]

\( \text{Net} \) applies \( \vec{F} = ma \) (as always!)

What \( F \) centripetal is required to keep \( m \) travelling in a circle of radius \( R \)?

(If we knew \( |\vec{a}| \) for circular motion, \( \text{Net} \) would give the answer.)

Dimensionally, \( [\vec{a}] = \left[ \frac{\Delta \vec{v}}{\Delta t} \right] = \frac{L}{T^2} = \frac{L}{T^2} \)

Proof: Assume \( a = v^2/R \), \( \Rightarrow \) Dimension \([a] = \text{Dimension}[v^2/R] \)

\[ \frac{L}{T^2} = \left( \frac{L}{T} \right)^2 \cdot \frac{L}{T^2} \]

And only \( p=2, q=-1 \) can give correct dimensionality for \( a \):

Therefore \( |\vec{a}| \propto v^2/R \), by dimensionality alone: \( |\vec{a}| = \text{const. } v^2/R \).

In fact \( |\vec{a}| = v^2/R \), as we can prove from definition:

\[ |\vec{a}| = \left| \frac{\Delta \vec{v}}{\Delta t} \right| = \lim_{t \to t_i} \frac{|\vec{v}_f - \vec{v}_i|}{t_f - t_i} \]

Enlarge:

\[ \Delta \vec{a} = \frac{1}{2} \Delta \vec{r} \]

\[ 2 |\Delta \vec{v}| = |\Delta \vec{a}| = 2 \left( \sin \frac{\Delta \theta}{2} \right) \Delta t \]

Also \( (\vec{v} \cdot \Delta \vec{t} = R \Delta \theta \) (if \( \theta \) is measured in radians)

And finally \( (\sin \theta \approx \frac{\Delta \theta}{2} \text{ (for small } \Delta \theta \text{ in radians}) \)

so that \( (\vec{a} \cdot \Delta t = 2vt \sin \frac{\Delta \theta}{2} \approx 2v \frac{\Delta \theta}{2} = v \Delta \theta \)

and \( |\vec{a}| = \left| \frac{\Delta \vec{v}}{\Delta t} \right| = \frac{v \Delta \theta}{\Delta t} = v/\frac{R}{R} \) \{using \( v/R = \frac{\Delta \theta}{\Delta t} \) from (c) \}.

\[ |\vec{a}| = v^2/R \]

\( \text{Centripetal } = m \vec{a} = mv^2/R \)