J. GRIFFIN 11/8/07

DILATION of Time for Moving Clocks

How it follows from c = constant = $3. \times 10^8$ m/sec, the same in EVERY INERTIAL FRAME, that $\Delta t = \gamma\ \Delta t'$ where Δt is the time interval in the Observer O's rest frame, S, and $\Delta t'$ is the time measured by a clock moving with constant velocity, V. (Note the the moving clock is at rest in another inertial frame S', whose observer, O', measures $\Delta t' = 2L/c$ for this same time interval.

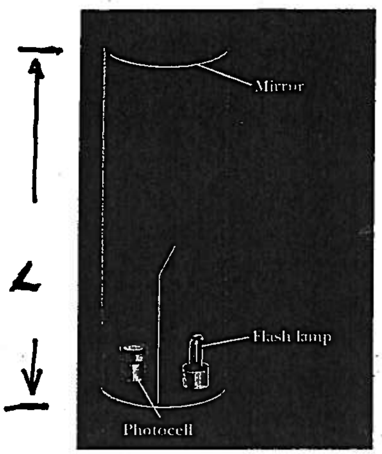

Figure 10-7 A light clock.

This clock ticks at time intervals of $\Delta t = 2L/c$, the time for the light pulse to travel from bottom to top & back.

FROM O's viewpoint at rest on S, the clock moves with speed V a distance $D = V\Delta t$ in time Δt. Then Fig 10.8 (below) shows that for O, the light pulse of moving clock travelled $2\sqrt{\frac{D^2}{4} + L^2} = 2H$ in time, Δt

Since light travels with speed c, & $D = V\Delta t$,

$$\underline{\Delta t} = 2H/c = \frac{2}{c}\sqrt{L^2 + (V\Delta t/2)^2}.$$

Then we square both sides to solve for Δt:

$$\Delta t^2 = \frac{4}{c^2}(L^2 + V^2\,\Delta t^2/4)$$

$$\Delta t^2(1 - V^2/c^2) = \frac{4L^2}{c^2} = \left(\frac{2L}{c}\right)^2 = (\Delta t')^2,$$

Since $2L/c$ is $\Delta t'$, the time per tick measured by O' with this clock, at rest in his frame, S'.

$$\text{Thus,}\quad \Delta t = \frac{1}{\sqrt{1 - V^2/c^2}}\,\Delta t' = \gamma\,\Delta t'$$

<u>O observes the moving clock to tick slower by the factor γ than a clock at rest in his frame.</u>

|← DISTANCE TRAVELLED in Δt is $D = V\Delta t$ →|

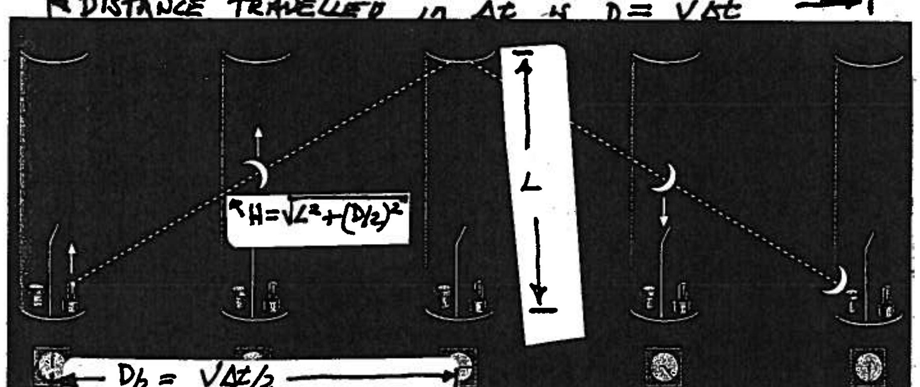

Figure 10-8 The light in the moving clock travels farther, and therefore the clock runs slower.

In fact, observer O in S observes this moving clock to tick once each $\Delta t = \gamma\Delta t'$, where $\gamma = 1/\sqrt{1 - v^2/c^2}$, always ≥ 1.

THUS <u>time</u> passes more slowly in a moving frame: E.g. living things age more slowly.