DILATION of Time for Moving Clocks

How it follows from \( c = \text{constant} = 3 \times 10^8 \text{ m/sec} \), the same in EVERY INERTIAL FRAME, that \( \Delta t = \gamma \Delta t' \)

where \( \Delta t \) is the time interval in the observer \( O \)'s rest frame \( S \),
and \( \Delta t' \) is the time measured by a clock moving with constant velocity, \( v \). (Note the the moving clock is at rest in another inertial frame \( S' \), whose observer, \( O' \), measures \( \Delta t' = 2\Delta \)

for this same time interval.)

From \( O' \)'s viewpoint at rest on \( S' \), the clock moves with speed \( v \) a distance
\( D = V \Delta t \) in time \( \Delta t \). Then Fig 10-8 (below) shows that for \( O \), the
light pulse of moving

\[ \sqrt{D^2 + L^2} = 2H \]

in time, \( \Delta t \).

Since light travels with speed \( c \), \( D = V \Delta t \),

\[ \Delta t = 2\Delta \]

Then we square both sides to solve for \( \Delta t' \):

\[ \Delta t'^2 = \frac{1}{\gamma^2} \left( \Delta^2 + \Delta' \right)^2 \]

\[ \Delta t' \Delta t = \frac{\Delta^2}{\gamma^2} = \frac{(\Delta t')^2}{(\Delta t)^2} \]

Since \( 2\Delta \) is \( \Delta t' \), the time per tick measured by \( O \) with this clock at rest in his frame \( S \).

Thus, \( \Delta t = \sqrt{1 - \frac{V^2}{c^2}} \Delta t' \), \( \gamma = \Delta t' / \Delta t \), \( \gamma \) always > 1.

An observer \( O \) in \( S \) observes this moving clock to tick once each \( \Delta t = \gamma \Delta t' \), when \( \gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} \), always > 1.

Thus time passes more slowly in a moving frame. E.g., living things age more slowly.