

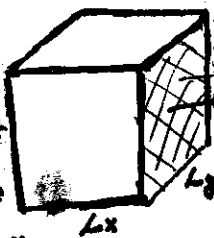
The Ideal Gas Law: $PV = nRT_A = NkT_A$,
 and the inference that $\frac{3}{2} kT_A = \langle KE_i \rangle = \text{Avg KE. of a gas molecule.}$
 can be obtained by elementary considerations, as follows.

Take the container to be a cube L on each side.

Assume that each gas particle has speed v and all velocity directions equally probable. Then

- (1) Compute the ^(AVERAGE) force exerted by the particles hitting the Right side of the cube & Divide this Avg. Force by $L^2 = \text{Area}$ of Right Face to obtain $P = |\vec{F}_{AV}| / A_R = |\vec{F}_R| / L_y L_z$, as follows.

- (2) To compute $|\vec{F}_{AV}|$, consider one molecule of gas, which has x-component of \vec{v} of $+v_x$ as it hits the RIGHT face. It rebounds elastically with a final x-component of velocity, $-v_x$. Its x-component of momentum is changed by $\Delta p_x = -mv_x - (+mv_x) = -2mv_x$, and this requires an impulse $\vec{F}_i \Delta t = -2mv_x$, where



\vec{F}_i is the force exerted by the wall on the i th molecule during the collision. And the force on the face is $\vec{F}_i = +2mv_x / \Delta t$, by N's III Law.

- (3) Compute the rate, R , at which molecules strike the right face, assuming that there are N molecules in the box. During a small interval Δt , all of the molecules within $v_x \Delta t$ of the right face which are travelling to the RIGHT (as $1/2$ are at any moment) will hit the face. Therefore $R \Delta t = \frac{N}{2} \cdot \frac{v_x \Delta t}{L}$ molecules hit the RIGHT face during Δt . [The fraction of such molecules is $\frac{R \Delta t}{N} = \frac{v_x \Delta t}{2L}$.]

- (4) The average force on the right face during a small interval Δt is the product of \vec{F}_i for one molecule times the No of molecules hitting during

$$\Delta t: F_{AV} = (\vec{F}_i) (N \text{ hits}) = \left(\frac{2mv_x}{\Delta t} \right) \left(\frac{N}{2} \frac{v_x \Delta t}{L} \right) = \frac{N}{L} m v_x^2$$

& The Pressure is $\frac{F_{AV}}{L^2} = P = \frac{1}{L_y L_z L_x} N m v_x^2 = \frac{1}{V} N (m v_x^2): PV = N (m v_x^2) = NkT_A$

- (5) In this way $kT_A = m v_x^2 = \frac{2}{3} \left[\frac{m}{2} (v_x^2 + v_y^2 + v_z^2) \right] = \frac{2}{3} \langle KE \rangle$; Thus

If the average $\langle KE \rangle$ of a molecule $\langle KE \rangle = \frac{3}{2} kT_A$, the IDEAL GAS LAW follows.

Recall $N = \text{no. of gas molecules in box.}$
 $n = \text{no. of moles of gas in box} = N/N_A$
 $R = \text{Boltzmann's gas constant per molecule}$
 $k = \text{gas constant per mole.}$