The Ideal Gas Law: \( PV = nRT_A = NkT_A \), and the inference that \( \frac{3}{2} kT_A = \langle KE \rangle = \frac{1}{2} m v^2 \) of a gas molecule can be obtained by elementary considerations, as follows.

Take the container to be a cube \( L \) on each side.

Assume that each gas particle has speed \( v \) and all velocity directions equally probable. Then

1. Compute the average force exerted by the particles hitting the right side of the cube & divide this Avg. Force by \( L^2 = \text{Area of Right Face} \) to obtain \( P = \frac{|F_{AVG}|}{A_R} = |F_R|/L^2 \).

2. To compute \( |F_{AVG}| \), consider one molecule of gas, which has x-component of \( \vec{v} \) of \(-v_x\) as it hits the right face. It rebounds elastically with a final x-component of velocity, \(-2v_x\). Its x-component of momentum is changed by \( \Delta p_x = -mv_{xf} - (-mv_x) = -2mv_x \).

3. This requires an impulse \( \vec{F} \cdot \Delta t = -2mv_x \), where \( \vec{F} \) is the force exerted by the wall on the \( i \)th molecule during the collision. And the force on the face is \( \vec{F}_i = +2mv_x/\Delta t \), by Newton's 3rd Law.

4. Compute the rate, \( R \), at which molecules strike the right face, assuming that there are \( N \) molecules in the box. During a small interval \( \Delta t \), all of the molecules within \( v_x \Delta t \) on the right face which are travelling to the RIGHT will hit the face. Therefore \( \frac{RAT = \frac{N}{L^2} v_x \Delta t}{N} \) molecules hit the right face during \( \Delta t \). [The fraction of fast molecules is \( \frac{RAT = \frac{N}{L^2} v_x \Delta t}{N} \).]

5. The average force on the right face during a small interval \( \Delta t \) is the product of \( \vec{F}_i \) for one molecule times the \( N/2 \) of molecules hitting during \( \Delta t \): \( F_{RAT} = (\vec{F}_i) \cdot (N/2 \Delta t) = \left( \frac{2mv_x}{L^2} \right) \left( \frac{N}{2} \right) v_x \Delta t = \frac{N}{2L^2} m v_x^2 \).

6. The Pressure is \( P = \frac{F_{RAT}}{L^2} = \frac{1}{L^2} \frac{N}{2} m v_x^2 = \frac{1}{2} \frac{N}{L^2} m v_x^2 = \frac{1}{2} N \langle m v_x^2 \rangle \); \( PV = N\langle m v_x^2 \rangle = kT \).

Thus \( \frac{3}{2} kT_A = \frac{1}{3} \langle m v_x^2 \rangle \) or \( \frac{3}{2} kT_A = \frac{1}{3} [m (v_x^2 + v_y^2 + v_z^2)] = \frac{2}{3} \langle KE \rangle \); Thus the average \( \langle KE \rangle \) of a molecule \( \langle KE \rangle = \frac{3}{2} kT_A \), the Ideal Gas Law follows.