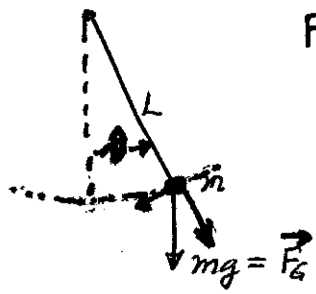
James J. Griffin

The Pendulum as a Simple Harmonic Oscillator (See p 301)



The driving force* is the force of gravity:

$F_G = (-mg\sin\vartheta, +mg\cos\vartheta) = (F_{G\perp}, F_{G\parallel})$,

where $\perp$ & $\parallel$ refer to the direction of the string (or rod) of the pendulum

We use $x = L\vartheta$ to measure the displacement of the pendulum from the vertical ($\vartheta = 0$) position.

Then $F_{G\perp} = -mg\sin\vartheta \approx -mg\,\vartheta \approx -mg\frac{x}{L}$, for small ϑ in radians**

since $\sin\vartheta \approx \vartheta$ & $\vartheta = \frac{\text{arc length}}{\text{radius}} = x/L$

Thus the restoring force is $F_{G\perp} = -\frac{mg}{L}\cdot x$ for small x

and resembles the Hooke's Law spring force, $F_{spring} = -kx$,

with (mg/L) replacing the spring constant, k.

Then the pendulum should be a SH Oscillator,

whose period is $T = 2\pi\sqrt{M/k} \rightarrow 2\pi\sqrt{m/\left(\frac{mg}{L}\right)} = 2\pi\sqrt{L/g}$.

This period { is independent of m, & depends only on L, the pendulum length, & g.

* Note that the force's $\perp$ component has a minus sign because it is directed in the direction <u>opposite</u> to the increasing ϑ direction.

** Also note that the small ϑ specification is <u>required</u>: If $\sin\vartheta \neq \vartheta$, as happens when ϑ gets too large, the the Force no longer resembles a Hooke's law force & the pendulum becomes "non-linear", and no longer behaves as a SHO.