Newton's Gravitation \( F_g = G M_s \frac{m_p}{R_{sp}^2} \)

(applied to planetary motion around Sun) and

centripetal acceleration for circular motion, \( \frac{v_p^2}{R_{sp}} \),

Combine to yield \[ \text{Kepler's IIIrd Law, } T_p^2 = (\text{const}) R_{ps}^3 \]

(a) \( v_p = \frac{2\pi R_{ps}}{T_p} \) relates speed of planet to its period, \( T_p \).

(b) \[ F_g = G M_s \frac{m_p}{(R_{sp})^2} = m_p \left( \frac{\pi^2}{4} \right) \frac{R_{ps}^2}{T_p^2} \]

\[ \frac{R_{ps}^2}{(G M_s)} = \frac{v_p^2}{4\pi^2} \Rightarrow T_p^2 = \left( \frac{4\pi^2}{G M_s} \right) R_{ps}^3 \quad (\text{Q.E.D.)} \]

Q.E.D. = "Quad Erat Demonstrandum" \( \Rightarrow " \) What had to be proven" (... nice phrase!)

A constant, \( (4\pi^2/G M_s) \), does not change from one planet to another.

Note that once the planet's distance from Sun is known, its period (i.e., its planetary year) is fixed.

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Kepler's laws can also be applied to EARTH Satellites in circular orbits... in which case M_s must be replaced by ME.

E.g., the height of a \textbf{Geosynchronous earth satellite} can be calculated from Kepler's III, as follows. (a) \( T = 24 \) hours for geosynchronous satellite \( (= 24 \times 3600 \text{ sec} = 8.64 \times 10^4 \text{ sec} = T) \). Thus,

(b) \[ R^3 = \left( \frac{G M_E}{4\pi^2} \right) T^3 \quad \Rightarrow \quad \left( \frac{R}{R_E} \right)^3 = \left( \frac{G M_E}{4\pi^2 R_E^3} \right) R_E^3 = \frac{3}{4\pi^2} \frac{T^2}{R_E^2} ; \text{ and} \]

(c) \[ \left( \frac{R}{R_E} \right) = \left( \frac{9T^2}{4\pi^2 R_E} \right)^{1/3} = \left( \frac{9 \times 24^2 \times 3600^2 \times 10^8}{4\pi^2 (6.37 \times 10^3)} \right)^{1/3} \]

\[ = \sqrt[3]{2.91 \times 10^2} = 6.6 \approx 6.6 \]

(d) i.e., \( R = R_E + h = 6.6 \) RE

\[ \frac{h}{R} \approx 5.6 \text{ RE} \]

** Just divide both sides by \( R^3 \), to confirm \( \frac{G M_E}{R_E^3} \).