J. Griffin 10/5/07

How N's Gravitation $F_G = G M_S m_P / R_{SP}^2$ (applied to planetary motion around sun) and centripetal acceleration for circular motion, v_P^2/R_{SP}, combine to yield $\boxed{\text{Keplers III}^{rd}\text{ Law}}$, $T_P^2 = (\text{const}) R_{PS}^3$.

(a) $v_P = \dfrac{2\pi R_{PS}}{T_P}$ relates speed of planet to its period, T.

(b) $F_G = G\dfrac{M_S \cancel{m_P}}{(R_{SP})^2} = m_P|\vec{a}_{cent}| = \dfrac{\cancel{m_P}\, v_P^2}{R_{SP}}$ .... NII w. F_G & circular motion

i.e. $\dfrac{GM_S}{R_{SP}} = v_P^2 = \dfrac{4\pi^2 R_{SP}^2}{T_P^2} \Rightarrow T_P^2 = \left(\dfrac{4\pi^2}{GM_S}\right) R_{SP}^3$ (Q.E.D.)*

* Q.E.D ≡ "Quod Erat Demonstrandum" ⟺ "What had to be proven" (.. nice phrase!)

& constant, $(4\pi^2/GM_S)$, does not change from one planet to another.

Note that once the planet's distance from sun is known its period (i.e. its planetary year) is fixed.

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Kepler's laws can also be applied to EARTH satellites in circular orbits... in which case M_S must be replaced by M_E.

E.g., the height of a $\boxed{\text{GEOSYNCHRONOUS earth satellite}}$ can be calculated from Kepler's III, as follows. (a) T = 24 hours for geosynchronous satellite (= 24 × 3600 sec = 8.64×10^4 sec = T). Thus,

(b) $R^3 = \left(\dfrac{GM_E}{4\pi^2}\right)T^2$ $\boxed{\text{OR}}$** $\left(\dfrac{R}{R_E}\right)^3 = \left(\dfrac{GM_E}{4\pi^2 R_E^2}\right)\dfrac{T^2}{R_E} = \dfrac{g}{4\pi^2}\dfrac{T^2}{R_E}$; and

(c) $\left(\dfrac{R}{R_E}\right) = \left(\dfrac{gT^2}{4\pi^2 R_E}\right)^{1/3} = \sqrt[3]{\dfrac{(9.8)\cdot(8.64)^2\times 10^8}{4\pi^2(6.37\times 10^6)}}\left(\dfrac{\cancel{m}}{\cancel{sec^2}}\cdot\dfrac{\cancel{sec^2}}{\cancel{m}}\right)^{1/3}$ dimensionless! (as it should be).

$= \sqrt[3]{2.91\times 10^2} = 6.63 \approx 6.6$

(d) I.e. $R = R_E + h = 6.6\, R_E$

$\boxed{\text{OR}}$ $h = 5.6\, R_E$

** Just divide both sides by R^3, to conjure up $g = \left(\dfrac{GM_E}{R_E^2}\right)$.