EXPONENTIAL GROWTH

... occurs whenever the increment/per unit time is proportional to the total size: $\frac{\Delta N}{\Delta t} = r \cdot N$, as e.g. in the doubling of the number of grains of wheat placed on each successive square of a chessboard (on p 128-129 of our text).

It characteristically occurs in living populations with adequate resources, but also in economic affairs, where, e.g., the amount returned on an investment per year is typically proportional to the size of the investment, which as a result, grows exponentially if the returns are continually reinvested.

If a number, $N(t)$, grows exponentially at a rate r per unit time, and if at $t=0$ it had the value N_0, then at time, t,

$$N(t) = N_0(e)^{rt} = N_0 \exp(rt) = N_0 \cdot (10)^{r \cdot t \cdot \log_{10}(e)}$$

where $e = 2.7183\ldots$ is the base of the natural system of logarithms & $\log_{10}(e) = 0.4343\ldots$ is its base-10 logarithm: $(10)^{\log_{10}(e)} = e = 2.7183$.

One remarkable feature of exponential growth is that it generates VERY LARGE numbers, if only one waits long enough.

A second remarkable feature is how sensitive that growth is to the rate, r, at which it occurs.

Two concrete examples of these are given on the following page: (a) The \$15,000 ice cream cone;

(b) Buying all of the WORLD'S ECONOMIES, and NOT just MANHATTAN for \$21.00.

They are based on a table showing how much \$1 will grow over different times at various growth rates.

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Exponential Growth: How $1 changes

...at various rates, r, over various times, t: $N = $1*e^{+rt}

Rate, r (in%/year):	2%/yr	4%/yr	6%/yr	12%/yr	20%/yr
Time,t (in years):					
1 yr	1.02	1.04	1.06	1.13	1.22
10 yr	1.22	1.49	1.82	3.32	7.39
20 yr	1.49	2.23	3.32	11.02	54.6
40 yr	2.23	4.95	11.02	121.5	2,981
100 yr	7.39	54.59	403.43	$1.63x10^5$	$4.85x10^8$
200 yr	54.59	$2,981	$\$1.63x10^5$	$\$2.65x10^{10}$	$2.35x10^{17}$
388 yr	$2,345	$\$5.5x10^6$	$\$1.29x10^{10}$	**$\$1.66x10^{20}$**	$5.03x10^{33}$
(= 1619->2007)					

Buying All of the World's Economies for $21, instead of just lower Manhattan

U.S. Gross National Product is $10 trillion = $\$10*(10^{+12}) = \$10^{+13}$
Then the total Capital Value of US is surely less than 1000XGNP < $\$10^{+16}$
But $21, which the Dutch are said to have paid for Lower Manhatten in 1619,
compounded at 12%/year for the 388 years since 1619. (rt = 46.68: $e^{+(rt)} = 1.66x10^{20}$.)
equals **$1*(1.66x10^{20})$** = $\$3.5 \times 10^{21}$ > 10^5 x Total Capital Value of US
This amount clearly exceeds the Capital Value of all of the world's economies.
Corollary: It is not possible to compound an investment at 12%/year for 3 centuries.

The $15,000 Ice Cream Cone

A $5 ice cream cone purchased on credit at an interest rate of 20%/year (or 1.7%/month as charged by some credit card accumulates to $5*(2,981) = $14,905 in 40 years. (rt = 0.20*40=8: e+(8) = 2,981). (In fact, the credit card company would not allow such a debt to accumulate, but would instead require minimum payments monthly. However, in an actual recent case (Wash.Post, 3/7/07, pD1) a person made credit card purchases of 3,200 in March, 2002. By February, 2007, he had paid $6,300 on the account but still owed $4000. This is equivalent to a compound interest rate of about 24% per year.}
Corollary: It is better to accumulate interest at 20% than to pay it.

{e is the natural number which is the base of the natural system of logarithms: e = 2.71828...;
The log of e in the commonly used base-10 system is 0.4343..., so that $e = 10^{+(0.4343)}$ =2.718
If (rt) =1, 2, 3, 4, ... the multiplication factors are e^1, e^2, e^3, e^4, ...= 2.7, 7.4, 20.1, 54.6...
and they are said to rise "exponentially", meaning very very rapidly, once they get large.