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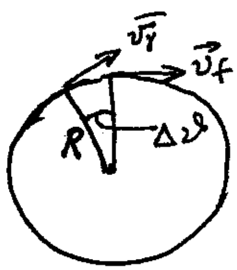
UNIFORM CIRCULAR MOTION



$|\vec{v}|$ = Speed = constant = $|v_f| = |v_i|$

R = Radius of Circle; m = mass of object.

N II applies $\vec{F} = m\vec{a}$ (as always!)

What $\vec{F}_{centripetal}$ is required to keep m travelling in a circle of radius R?

(If we knew $|\vec{a}|$ for circular motion, N II would give the answer.)

Dimensionally, $[\vec{a}] = \left[\frac{\Delta\vec{v}}{\Delta t}\right] = \frac{L}{T}\cdot\frac{1}{T} = \frac{L}{T^2}$

Assume $a = v^p R^q$, $\Rightarrow$ Dimension $[a]$ = Dimension$[v^p R^q]$

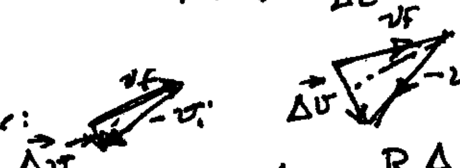
$\frac{L}{T^2} = \left(\frac{L}{T}\right)^p \cdot L^q$

And only $p=2$, $q=1$ can give correct dimensionality for a: $|\vec{a}|$ = const. v^2/R.

Therefore $|\vec{a}| \propto v^2/R$, by dimensionality alone.

In fact $|\vec{a}| \equiv v^2/R$, as we can prove from definition of $|\vec{a}| = \frac{|\Delta\vec{v}|}{\Delta t} = \lim_{t_f \to t_i} \frac{|\vec{v}_f - \vec{v}_i|}{t_f - t_i}$ (See sketch above) (a)

Enlarge:



$2\frac{|\Delta\vec{v}|}{2} = |\Delta\vec{v}| = 2\left(\sin\frac{\Delta\vartheta}{2}\right)|v|$ (b)

(if ϑ is measured in radians)

Also (c) $v\Delta t = R\Delta\vartheta$ (for small $\Delta\vartheta$ in radians)

And finally (d) $\sin\frac{\Delta\vartheta}{2} \approx \frac{\Delta\vartheta}{2}$

so that (b) $|\Delta\vec{v}| = 2v\sin\frac{\Delta\vartheta}{2} \approx 2v\cdot\frac{\Delta\vartheta}{2} = v\Delta\vartheta$

and $|\vec{a}| = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{v\Delta\vartheta}{\Delta t} = v\left(\frac{v}{R}\right)$, {using $v/R = \frac{\Delta\vartheta}{\Delta t}$ from (c)}.

$|\vec{a}| = v^2/R$

$$\boxed{F_{centripetal} = m\vec{a} = m v^2/R}$$