

Physics 117

Homework Solutions to HW Set # 8

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By	<u>QJ</u>
On	___/___/___

CH 10: CQ 23, 33, 34; Ex 7, 13, 17

CH 11: CQ 5, 9; Ex 3, 8

Ch 10 CQ 23. To an observer in the lab frame, the time interval measured by a clock (or the average lifetime of a decaying particle) moving through the lab at speed, v , is dilated (i.e., increased) by the adjustment factor, $\gamma = (1 - v^2/c^2)^{-1/2} \geq 1$, as compared with a clock at rest in the lab.

Then if the muon lifetime at rest (i.e. observed in its own rest frame) is τ_{μ}^0 , the lab observer will measure this lifetime to be $\tau_{\mu}^v = \gamma \tau_{\mu}^0 \geq \tau_{\mu}^0$. Thus the lab lifetime of $8 \mu\text{s}$ (μs = micro-seconds, and this μ , - for "micro" = 10^{-6} has nothing to do with the name and symbol, μ , for the mu-meson, or muon.) is greater than that of the muon at rest by the factor $\gamma \geq 1$, and $\tau_{\mu}^0 \leq \tau_{\mu}^v$. I.e. muon's lifetime at rest is less than its lifetime measured in a lab through which it moves.

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By	<u>[Signature]</u>
On	<u>3/15/04</u>

Ch 10 CQ 33. If $T_{\mu}^0 = 2.2 \mu\text{s}$ is the average lifetime of a muon at rest (i.e. as measured by an observer in its own rest frame), then at a speed c (and no speed can be greater!), it could travel $2.2 \times 10^{-6} \times 3 \times 10^8 \text{ m/s} = 660 \text{ m}$, at most, before decaying.

But if the muon is moving in a lab frame on earth, then its lifetime is dilated by a factor of γ , and the maximum distance would be $c \cdot \gamma T_{\mu}^0 \gg c T_{\mu}^0 = 660 \text{ m}$. In this case the observer must be in the reference system through which the muon is moving (e.g. earth's lab frame).

An observer in the muon rest frame would report
 (a) that muon is at rest, and earth is moving with velocity $-\vec{v}$, just opposite to what earth's observer says about muon, and

(b) that the distance from upper atmosphere to earth's surface, which earth/lab measures to be D is actually "contracted" to the smaller distance $D/\gamma = D'$ for the observer at rest on the muon. He says that the small earth-atmosphere distance D' travels past the muon at speed

$$\frac{D'}{T_{\mu}^0} = v_E = \frac{D}{\gamma T_{\mu}^0}, \text{ since } D' = \frac{D}{\gamma} \text{ is a contracted moving length.}$$

The earth lab observer says that the muon is moving

$$\text{with speed } v_{\mu} = \frac{D}{T_{\mu}^0} = \frac{D/\gamma}{T_{\mu}^0}, \text{ given in his}$$


frame by the distance D (at rest & uncontracted) divided by the dilated lifetime of the moving muon $T_{\mu}^0 = \gamma T_{\mu}^0$

The two observers agree only on the value of $|v|$.

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By	
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Ch 10 CQ 34. If the muon's arrival at earth's surface after traversing $D \gg 660 \text{ m}$ is to be explained in terms of length contraction, then the distance travelled must be moving in the frame of the observer. If D is the height of the top of the earth's atmosphere in the earth's rest frame and is to be moving, then the observer is in the rest frame of the moving muon. He measures objects moving in his own rest frame to be shortened (contracted) in the direction parallel to their velocity by a factor, $1/\gamma$. Then he measures the height D to be a much smaller value $D' = D/\gamma \ll 660 \text{ m}$, and says that it can pass by the muon in the span of its lifetime in its rest frame, T_μ^0 .

To the earth lab observer D is at rest, so no question of contraction arises. On the other hand the muon is moving, so its internal clock (which defines its average lifetime) shows a time dilation. He explains the great distance travelled (D) by his observation that the muon's average lifetime is $\gamma T_\mu^0 = T_\mu^v \gg T_\mu^0$.

Thus the two observers disagree both about clock times & distances ... they agree only on the relative speed $v = \frac{D'}{T_\mu^0} = \frac{D}{\gamma T_\mu^0} = \frac{D}{T_\mu^v}$.

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By	<u>[Signature]</u>
On	<u>3/15/04</u>

Ch 10 Ex 7.

$$\tau_{\pi}^v = \gamma \tau_{\pi}^0 = (1 - v^2/c^2)^{-1/2} \tau_{\pi}^0$$

$$2.69 \times 10^{-9} \text{ s} = \frac{2.69 \times 10^{-9}}{7.09} = \tau_{\pi}^0 = 3.79 \times 10^{-10} \text{ sec}$$

$\tau_{\pi}^0 = 3.79 \times 10^{-10} \text{ sec}$
 = lifetime of pion at rest

Ex 13 Exactly the same answer since she is observing the train in the rest frame of the train. An observer in the station's rest frame, however, would measure the moving train's length to be contracted by a factor $1/\gamma$ (≤ 1). Then she again measures 200 m

Ex 17. Since $p^R = \gamma m v$ is the relativistic momentum (replacing $p^{NR} = m v$, non relativistically) we have $F_R \Delta t = \text{impulse} = \text{change in momentum} = p_f^R - p_i^R$
 and $F_R \Delta t = \gamma_f m v_f - 1 \cdot m \cdot 0 = \gamma_f m v_f$.

The corresponding non-relativistic result is $F_{NR} \Delta t = 1 \cdot m v_f$.

Since Δt is the same for both $F_R = \frac{\gamma_f \cdot m v_f}{\Delta t} = \gamma_f \cdot F_{NR}$

The relativistic force must be larger by the factor γ :

$$\gamma = [1 - (1 - 10^{-4})^2]^{-1/2} = (1 - 1 + 2 \cdot 10^{-4} - 10^{-8})^{-1/2} = \sqrt{1/2 \times 10^{-4}}$$

$$= \frac{10^2}{\sqrt{2}} = 70.7$$

Therefore, $F_R = (9.5)(70.7) = 672 \text{ N}$

Note that calculator rounding error can lead to false results when the answer gets too small. Then writing $v/c = 1 - \epsilon$ yields $(1 - v^2/c^2)^{-1/2} = [1 - (1 - \epsilon)^2]^{-1/2} = [1 - (1 - 2\epsilon + \epsilon^2)]^{-1/2} \approx \frac{1}{\sqrt{2\epsilon}}$ for small ϵ .
 since $\epsilon^2 \ll \epsilon$ when $\epsilon \ll 1$.

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By
On <u>3/15/04</u>

Ch 11: CG 5, 9 / Ex 3, 8.

11: Q5) Additional examples in which experimental results agree with a model strengthen our belief in it. However, a model can never be proven true.

11: Q9) Salt, granite and water ^{are} not elements. Each is composed of more than one element.

11: Ex3) Only 4g of hydrogen is required to use up 32g of oxygen fully \therefore Amount of water that can be formed = $4(1g) + 32g = \boxed{36g}$, and 4g of H will be left over.

11: Ex8) Molecular mass of sulphur = 32 amu.

\therefore 1 mole of sulfur weighs 32g.

There are Avogadro number of molecules in one mole: $N_A = 6 \times 10^{23}$.

\therefore 32g of sulfur contain $N_A = 6.02 \times 10^{23}$ molecules.

1g. of sulfur contains $\frac{6.02 \times 10^{23}}{32}$ molecules

$$= \boxed{1.88 \times 10^{22}} \text{ molecules of S.}$$

- END of Solution #8 -