

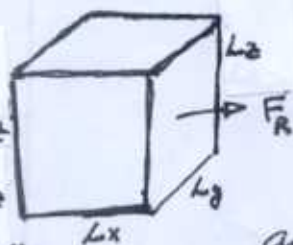
The Ideal Gas Law:  $PV = nRT_A = NkT_A$ , and the inference that  $\frac{3}{2} kT_A = \langle KE_i \rangle = \text{Avg KE. of a gas molecule.}$  can be obtained by elementary considerations, as follows.

Take the container to be a cube  $L$  on each side.

Assume that each gas particle has speed  $v$  and all velocity directions equally probable. Then

- (1) Compute the <sup>(AVERAGE)</sup> force exerted by the particles hitting the Right side of the cube & Divide this Avg. Force by  $L^2 = \text{Area of Right Face}$  to obtain  $P = |\vec{F}_{AV}| / L^2 = |\vec{F}_R| / L_y L_z$ , as follows.

- (2) To compute  $|\vec{F}_{AV}|$ , consider one molecule of gas, which has x-component of  $\vec{v}$  of  $+v_x$  as it hits the RIGHT face. It rebounds elastically with a final x-component of velocity,  $-v_x$ . Its x-component of momentum is changed by  $\Delta p_x = -mv_x - (+mv_x) = -2mv_x$  and this requires an impulse  $\vec{F}_i \Delta t = -2mv_x$ , where



$F_i$  is the force exerted by the wall on the  $i$ th molecule during the collision. And the force on the face is  $\vec{F}_i = +2mv_x / \Delta t$ , by N's III law.

- (3) Compute the rate,  $R$ , at which molecules strike the right face, assuming that there are  $N$  molecules in the box. During a small interval  $\Delta t$ , all of the molecules within  $v_x \Delta t$  of the right face which are travelling to the RIGHT (as  $1/2$  are at any moment) will hit the face. Therefore  $R \Delta t = \frac{N}{2} \cdot \frac{v_x \Delta t}{L}$  molecules hit the RIGHT face during  $\Delta t$ . [The fraction of such molecules is  $\frac{R \Delta t}{N} = \frac{v_x \Delta t}{2L}$ .]

- (4) The average force on the right face during a small interval  $\Delta t$  is the product of  $\vec{F}_i$  for one molecule times the No of molecules hitting during

$\Delta t: \vec{F}_{AV} = (\vec{F}_i) (N \text{ hits}) = \left( \frac{2mv_x}{\Delta t} \right) \left( \frac{N}{2} \frac{v_x \Delta t}{L} \right) = \frac{N}{L} m v_x^2$   
 & The Pressure is  $\frac{F_{AV}}{L^2} = P = \frac{1}{L_y L_z} N m v_x^2 = \frac{1}{V} N (m v_x^2): PV = N (m v_x^2) = nRT_A$

- (5) In this way  $kT_A = m v_x^2 = \frac{2}{3} \left[ \frac{m}{2} (v_x^2 + v_y^2 + v_z^2) \right] = \frac{2}{3} \langle KE \rangle$ ; Thus  
 If the average  $\langle KE \rangle$  of a molecule  $\langle KE \rangle = \frac{3}{2} kT_A$ , the IDEAL GAS LAW follows.

Recall  $\left\{ \begin{array}{l} N = \text{no of gas molecules in box.} \\ n = \text{no. of moles of gas in box} = N/N_A \\ R = \text{Boltzmann's gas constant per molecule} \\ R = N_A k = \text{gas constant per mole.} \end{array} \right.$