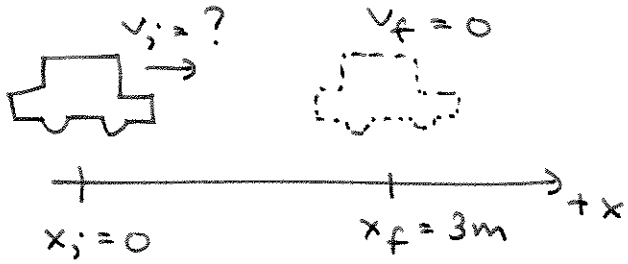
Homework #5

① $\Delta v = v_f - v_i = -3\ m/s$



$\Delta v = a\,\Delta t \qquad a = 1.5g \simeq 15\ m/s^2$

$\Rightarrow -v_f = 15\,\Delta t$

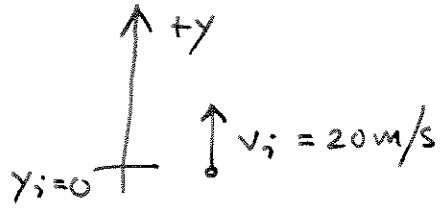
$$\boxed{\Delta t = -\frac{1}{15} v_f}$$

$$\Delta x = v_i \Delta t + \frac{1}{2} a \Delta t^2$$

$$3\ m = v_i\left(-\frac{1}{15} v_i\right) + \frac{1}{2}(15)\left(-\frac{1}{15} v_i\right)^2$$

$$3 = \frac{1}{30} v_i^2 \quad \Rightarrow \quad \boxed{v_i = 9.5\ m/s}$$

② (a) $\Delta y = v_i \Delta t - \frac{1}{2} g \Delta t^2 \qquad g \simeq 10\ m/s^2$



$$+5\ m = (20\ m/s)\,\Delta t - (5\ m/s^2)\,\Delta t^2$$

$$5\Delta t^2 - 20\Delta t + 5 = 0$$

$$\Rightarrow \Delta t = \frac{1}{10}\left[+20 \pm \sqrt{(-20)^2 - 4 \cdot 5 \cdot 5}\right]$$

$$= \begin{cases} 0.27\ s \\ 3.7\ s \end{cases}$$

$$v_f = v_i - g\Delta t = \begin{cases} +17\ m/s & \Delta t = 0.27\ s \\ -17\ m/s & \Delta t = 3.7\ s \end{cases}$$

$\Rightarrow$ same speed on the way up ($v_f > 0$) and on the way down ($v_f < 0$)

(b) $\Delta y = -5\,m \Rightarrow$

$$-5\,m = (20\,m/s)\,\Delta t - (5\,m/s^2)\,\Delta t^2$$

$$5\,\Delta t^2 - 20\,\Delta t - 5 = 0$$

↑ only difference from previous case

$$\Delta t = \frac{1}{10}\left[+20 \pm \sqrt{(-20)^2 - 4 \cdot 5 (-5)}\right]$$

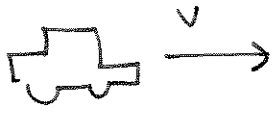
$$= \begin{cases} -0.24\,s \\ +4.2\,s \end{cases} \leftarrow \text{negative } \Delta t \text{ means } \underline{\text{before}}\ t = 0$$

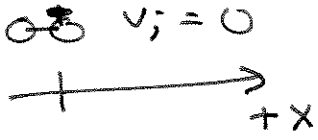
$$v_f = v_i - g\,\Delta t = \begin{cases} 22\,m/s \\ -22\,m/s \end{cases} \Rightarrow \text{going up at } t = -0.24\,s$$

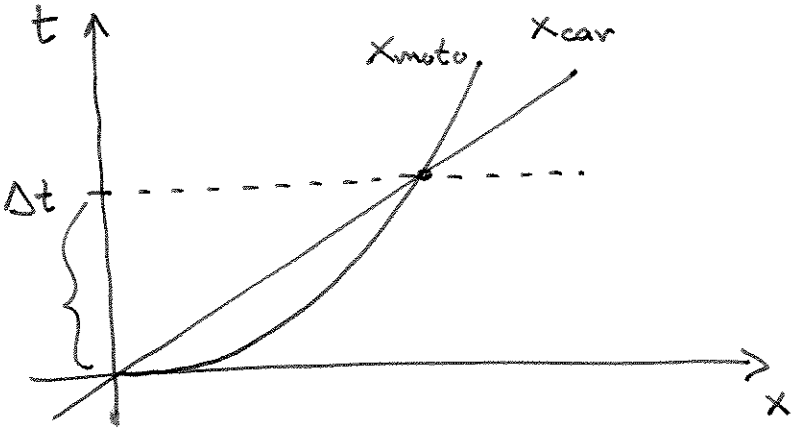
Equations assume ball is in free fall for all times $\Rightarrow$ going up for $t < 0$.

(3) (a) car: $x_i = 0$ $t_i = 0$

motorcycle: $x_i = 0$ $t_i = 0$







Δt = time to catch car

(b) $\Delta x_{car} = v_{car}\,\Delta t$

$\Delta x_{moto} = \frac{1}{2} a_{moto}\,\Delta t^2$ $\quad (v_{i,moto} = 0)$

Catches car when $\Delta x_{moto} = \Delta x_{car}$

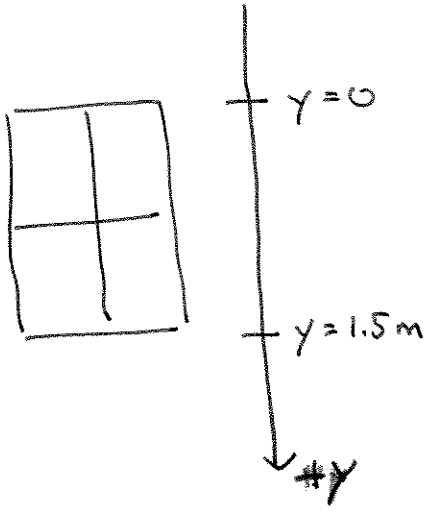
$\Rightarrow v_{car}\,\Delta t = \frac{1}{2} a_{moto}\,\Delta t^2$

$\Rightarrow \Delta t = 0$ or $\Delta t = \dfrac{2 v_{car}}{a_{moto}} = 7.9\ s$

$\Delta t = 0$: soln where car and motorcycle are initially at same position

(c) $\Delta x_{car} = v_{car}\,\Delta t = (55\ m/s)\cdot(7.9\ s) = 430\ m$

④ (a) From top of window to bottom of window



$y_i = 0$ $\quad y_f = 1.5\ m$

$v_i = ?$ $\quad \Delta t = 0.3\ s$

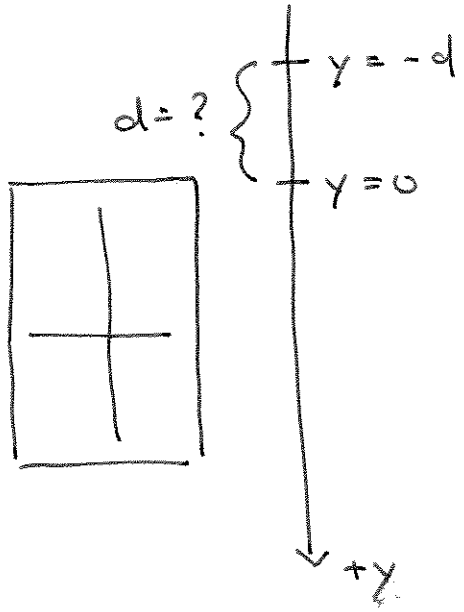
$\Delta y = v_i\,\Delta t + \frac{1}{2} g\,\Delta t^2$

$(1.5\ m) = v_i\,(0.3\ s) + \frac{1}{2}(9.8\ m/s^2)(0.3\ s)^2$

$\Rightarrow (0.3)\,v_i = 1.5 - 0.44 = 1.06$

$\boxed{v_i = 3.5\ m/s}$

(b) From dropped position to top of window



$y_i = -d = ?$ $\quad y_f = 0$

$v_i = 0$ $\quad v_f = 3.5\ m/s$

$\Delta t = ?$

$$\Delta v = g\,\Delta t$$

$$3.5\ m/s = (9.8\ m/s^2)\,\Delta t$$

$$\Rightarrow \boxed{\Delta t = 0.36\ s}$$

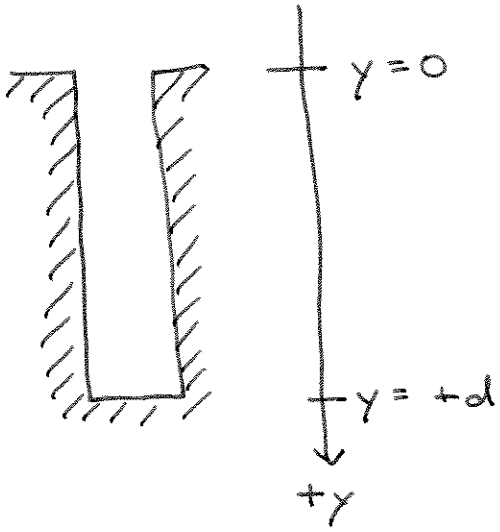
$$\Delta y = \frac{1}{2} g\,\Delta t^2$$

$$d = \frac{1}{2}(9.8\ m/s^2)(0.36\ s)^2 = \underline{\underline{0.63\ s}}$$

(5) $2\ s = \Delta t_{drop} + \Delta t_{sound}$

Δt_{drop} = time for stone to drop

Δt_{sound} = time for sound to reach top of hole



Dropped stone:

$v_i = 0 \Rightarrow \Delta y = \frac{1}{2} g\,\Delta t_{drop}^2$

$$d = \frac{1}{2}(9.8\ m/s^2)\,\Delta t_{drop}^2$$

$$\Rightarrow \Delta t_{drop} = 0.45\sqrt{d}$$

Sound:

$$\Delta y = v_{sound}\,\Delta t_{sound}$$

$$-d = (-300\ m/s)\,\Delta t_{sound}$$

$$\boxed{\Delta t_{sound} = +\frac{1}{300}d}$$

$$\Rightarrow 2 = 0.45\sqrt{d} + \frac{1}{300}d$$

Quadratic equation in $X = \sqrt{d}$

$$\frac{1}{300}X^2 + (0.45)X - 2 = 0$$

$$\Rightarrow X = 300\left[-0.45 \pm \sqrt{(-0.45)^2 - 4\cdot\frac{1}{300}\cdot(-2)}\right]$$

$$= \begin{cases} +8.6 \\ -250 \leftarrow \text{discard:} \end{cases}$$

$X > 0$

$$d = X^2 = (8.6)^2$$

$$\boxed{d = 74\ m}$$