Principle of Gravity Gradiometry

Gravity is described by a potential $\phi(x_i, t)$, which is an unobservable. Its first spatial derivatives form a vector: $g_i \equiv -\partial\phi/\partial x_i$. This gravitational field is indistinguishable from a platform acceleration by the Equivalence Principle. In General Relativity, the intrinsic field that uniquely characterizes the gravity field is the Riemann curvature tensor. The corresponding quantity in Newtonian gravity is the "gravity gradient" tensor:

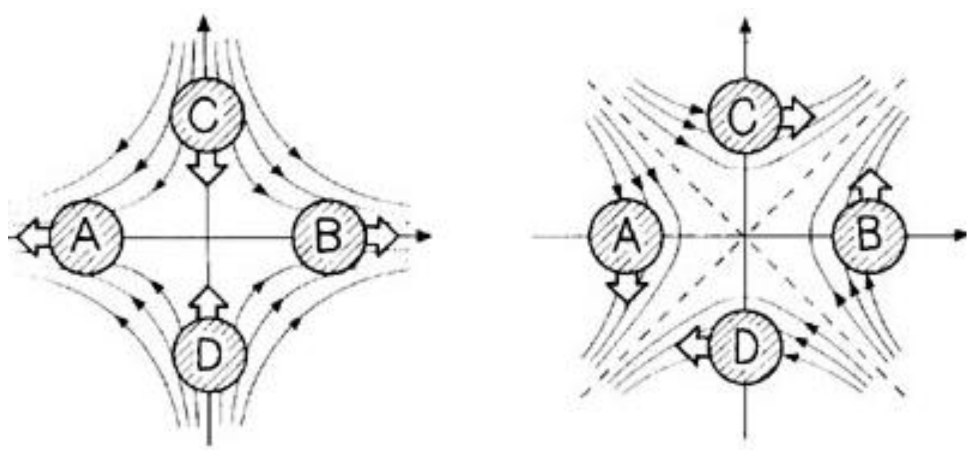
$$\Gamma_{ij} \equiv \partial^2\phi / \partial x_i \partial x_j. \tag{1}$$

The gravity-gradient tensor is symmetric. Its trace is related to the local mass density by Poisson's equation, an expression of the $1/r^2$ law:

$$\sum_i \Gamma_{ii} = \nabla^2\phi = 4\pi G\rho. \tag{2}$$

In free space (ρ = 0), this trace must vanish. This leaves only *five* independent components for the gravity-gradient tensor: two diagonal ("in-line") and three off-diagonal ("cross") components.

An in-line-component gradiometer can be constructed by differencing signals between two linear accelerometers whose sensitive axes are aligned along their separation, as shown in Figure 1(a). A cross-component gradiometer can be constructed by combining signals from four



(a) (b)

Figure 1. (a) An in-line-component gradiometer can be constructed by differencing linear accelerations between two masses *A* and *B*, or *C* and *D*. (b) A cross-component gradiometer can be constructed by combining linear accelerations on four masses *A*, *B*, *C*, and *D*, or by differencing angular accelerations between two rotating arms *AB* and *CD*.

test masses with their sensitive axes as indicated in Figure 1(b) or by differencing signals between two concentric angular accelerometers whose arms are orthogonal to each other. A *tensor* gradiometer could be constructed by combining three in-line-component gradiometers with three cross-component gradiometers.

The gravity gradient measured in a rotating reference frame is related to the gradient in the inertial frame by

$$\mathbf{\Gamma}' = \begin{pmatrix} \Gamma_{11} + (\Omega_2^2 + \Omega_3^2) & \Gamma_{12} - \Omega_1\Omega_2 & \Gamma_{13} - \Omega_1\Omega_3 \\ \Gamma_{21} - \Omega_2\Omega_1 & \Gamma_{22} + (\Omega_3^2 + \Omega_1^2) & \Gamma_{23} - \Omega_2\Omega_3 \\ \Gamma_{31} - \Omega_3\Omega_1 & \Gamma_{32} - \Omega_3\Omega_2 & \Gamma_{33} + (\Omega_1^2 + \Omega_2^2) \end{pmatrix}, \tag{3}$$

where $\mathbf{\Omega} = \{\Omega_1,\ \Omega_2,\ \Omega_3\}$ is the angular rate vector in the gradiometer coordinate system. Thus centrifugal acceleration constitutes a very important error source. This error is especially troublesome since wideband platform jitters are downconverted to the low-frequency measurement bandwidth by its nonlinear characteristic.

Gravity gradiometers are usually operated in a dynamically noisy environment to detect very weak differential signals. Passive vibration isolation is not possible, unlike in a gravitational-wave detector, because the signal frequencies for the gradiometry are very low in general ($\leq$ 0.1 Hz). Active isolation, or "platform stabilization," can be applied to the angular degrees of freedom. It is difficult, however, to achieve active isolation in all six degrees of freedom. Therefore, it is important to build into the device as high common-mode-rejection ratios as possible for both linear and angular accelerations.

In order to reject the common mode along the sensitive axis, the scale factors of the component accelerometers must be matched precisely. This, in turn, requires high stablility of the scale factors. This is where a cryogenic instrument has another important advantage. A superconducting gravity gradiometer (SGG) has *extremely stable* scale factors due to the increased mechanical stability of materials at low temperatures and the ultrahigh stability of persistent currents [Chan and Paik, 1987; Chan, Moody, and Paik, 1987; Moody, Canavan, and Paik, 2002].

The gradiometer will be sensitive to the transverse component of the acceleration by misalignment of the sensitive axes. In an in-line-component gradiometer, linear and angular accelerations of the platform,

$\vec{a}$ and $\vec{\alpha}$, couple to the differential mode through departures from parallelism and concentricity of the sensitive axes of the component accelerometers, $\delta\hat{n}$ and $\delta\hat{\ell}$, respectively [Chan and Paik, 1987]:

$$\delta\Gamma_a = \frac{1}{\ell}\delta\hat{n} \cdot \vec{a}, \tag{4}$$

$$\delta\Gamma_\alpha = \left(\delta\hat{\ell} \times \hat{n}\right) \cdot \vec{\alpha}. \tag{5}$$

There are corresponding error sources in a cross-component device. A departure of the rotation axes from parallelism provides a coupling mechanism for angular acceleration, whereas an asymmetric mass distribution in each moment arm (a departure of the rotation axis from the mass symmetry axis) causes linear acceleration to couple to the gradiometer.

For an SGG, the enhanced mechanical stability of materials at low temperatures guarantees that $\delta\hat{n}$ and $\delta\hat{\ell}$ are also stable. These error coefficients can be measured definitively during the initial setup, multiplied by the linear and angular accelerations of the gradiometer platform, and subtracted from the gradiometer output. By applying this "residual common-mode balance" [Moody, Chan, and Paik, 1986], $\delta\hat{n}$ and $\delta\hat{\ell}$ can be effectively reduced to very small values.

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