

FIG. 1.  ${}^4\text{He}^+/\text{U}$  rocking curve for the U surface peak near the  $\langle 110 \rangle$  direction on the  $\text{UO}_2(111)$  surface. Experimental data are individual points with error bars. Solid curve (—): ideal bulk termination, no surface vibrational enhancement or correlation. Dashed curve (---): 0.19 Å outward relaxation of the U surface layer, surface vibrational enhancement of 1.2 and no correlation.

like oxygen overlayer residing on an undistorted, underlying bulklike U lattice.<sup>4</sup> The present study addresses the possibility of U lateral distortions on this surface. Preliminary rocking curve data for the U surface peak, taken near the  $\langle 100 \rangle$  direction for a  $\text{UO}_2(100)$  vicinal surface [ $4.5^\circ$  from  $(100)$ ], show a symmetrical curve  $\sim 0.75$  atoms/string larger than that expected for a simple bulk termination. Both surface vibrational enhancement and lateral distortion of U terrace atoms can be used to fit the data, although step effects may also play a role in the larger surface peak.

In summary, we have used RBS and Monte Carlo simulations to demonstrate an outward U surface layer relaxation for the  $\text{UO}_2(111)$  surface.

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## Summary Abstract: Relationship between many-parameter lattice gas systems and simpler models: Easy approximations for $T_c$

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While lateral interactions between chemisorbed atoms markedly affect many properties of the overlayer, they resist quantitative measurement because of their small size relative to binding energies and diffusion barriers. A powerful way to probe the interactions is to measure the 2- $d$  phase diagram of the overlayer. The lateral interactions can then be treated as adjustable parameters in theoretical calculations (most often Monte Carlo) of the phase diagram.<sup>1-5</sup> In this procedure it is typically necessary to limit the number of parameters (and values for each). Thus, it would be useful to have a method, given  $T_c$  for one (typically small) set of interactions, to estimate  $T_c$  for a system of the same symmetry for a different (typically larger) set. We have found such a scheme<sup>2,4</sup> which is remarkably accurate for simple lattice gases for a wide choice of lateral interactions.

We describe the use of this method in terms of a  $c(2 \times 2)$  overlayer on a square lattice, as shown in Fig. 1. A  $c(2 \times 2)$  overlayer forms when there is a repulsive first neighbor interaction energy  $E_1$ . If this is the only interaction, the problem translates directly into the zero field Ising model, for which  $E_1 \cong 1.76 kT_c$ ,<sup>6</sup> shown as Onsager solution in Fig. 1. However, in real systems there are also longer range interactions, for instance, a second neighbor interaction  $E_2$ , as shown in Fig. 1. To determine the change in  $T_c$  caused by the addition of  $E_2$ , we consider the minimum energy required for an atom to move from an ordered site. As indicated by the  $\times$ 's in Fig. 1, the central atom can disorder most easily into a nearest neighbor site. If  $E_2 = 0$ , the energy cost for this move is  $3E_1$ , since there is no repulsion from the newly formed vacancy at the "central" site. (In contrast, in mean field the-

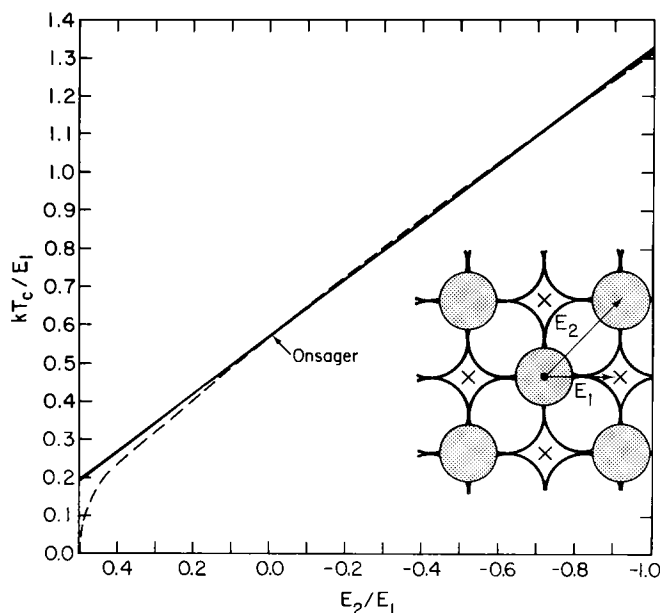


FIG. 1. Inset:  $c(2 \times 2)$  overlayer on a square lattice of adsorption sites. The lowest energy excitation for the central atom is into one of the four sites marked by  $\times$ 's. Graph: Transition temperature as a function of the second neighbor interaction energy for a  $c(2 \times 2)$  overlayer on a square lattice. Solid line: Eq. (1). Dashed line: Transfer matrix scaling calculation (essentially exact).

ory the adatom goes to a random vacancy with initial excitation energy  $4E_1$ . If  $E_2 \neq 0$ , the energy cost is  $3E_1 - 4E_2$ . We then scale  $T_c$  using these excitation energies as follows:

$$\frac{3E_1 - 4E_2}{T_c(E_2)} = \frac{3E_1}{T_c(0)}$$

or

$$T_c(E_2) = T_c(0) \left( 1 - \frac{4}{3} \frac{E_2}{E_1} \right). \quad (1)$$

This approximation is plotted in Fig. 1 along with essentially exact values obtained by transfer matrix scaling<sup>7</sup> calculations.

Equation (1) works extremely well in the range  $-1.0 \leq E_2/E_1 \leq 0.1$ . The mechanisms of its failure outside this range illustrate important restrictions on the use of the excitation scheme to predict  $T_c$ . First consider the case where  $E_2/E_1 \geq 1/2$ . Here the  $c(2 \times 2)$  structure with ground state energy  $4E_2$  per adatom becomes unstable with respect to a  $(2 \times 1)$  structure with ground state energy  $2E_1$  per atom.

Clearly, to use the excitation scheme the added interactions must not change the symmetry of the overlayer. In the range  $0.1 < E_2/E_1 < 1/2$  the impending symmetry change at  $E_2/E_1 = 1/2$  complicates the prediction of the minimum energy required to move an atom (i.e., multiatom moves evidently become important) causing the approximation to fail. The excitation scheme for  $E_2$  strongly attractive ( $-E_2 \gg E_1$ ) fails for a different reason. In this case the nearest neighbor repulsion becomes unimportant compared to the second neighbor attraction, and disordering into one of the sites marked by  $\times$ 's is not favored over disordering into a random site. In short, the excitation scheme can be used only if the minimum-energy disordering move is significantly lower in energy than any other possible move.

The excitation scheme can be used for lattice gas systems with a variety of symmetries and hence with different sets of interactions than discussed here, for instance, the  $(\sqrt{3} \times \sqrt{3}) R 30^\circ$  and  $(2 \times 2)$  overlayers on a triangular lattice, the  $(2 \times 2)$  on a honeycomb lattice, and the  $(2 \times 1)$  on a bcc (110) lattice. In addition, it can be used to estimate the effects of trio (non-pairwise) interactions<sup>8</sup> on the phase diagram. A discussion of these applications, detailing their accuracy and their range of reliability, will be presented elsewhere.<sup>9</sup>

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